First project: Due on Friday, May 14

Show that the problem

\[
\begin{cases}
-u''(x) + u(x) = f(x), & 0 < x < 1, \\
u(0) = 0, & u'(1) = 0
\end{cases}
\]

can be given a weak variational formulation.

Formulate a FEM for this problem using piecewise-linear functions. Determine the corresponding linear system of equations in the case of a uniform partition and study in particular how the boundary condition \( u'(1) = 0 \) is approximated by the method.

Write a computer program for the piecewise-linear FEM applied to this problem. Present calculations with \( h = 0.1 \) and \( h = 0.2 \), for \( f(x) = 2 + 2x - x^2 \). Approximate \( \int f(x) \varphi_j(x) dx \) by the midpoint rule. (the exact solution is \( u(x) = x(2 - x) \)).

(see also section 1.8: Remarks on programming)