Math 155, Vese

Homework # 8, due on Friday, June 2

[1] Start with equation (5.4-19) and derive equation (5.4-21).

[2] Consider the motion blur in the frequency domain given by

$$H(u,v) = \int_0^T e^{-2\pi i[ux_0(t)+vy_0(t)]}dt.$$

For uniform motion given by $x_0(t) = \frac{at}{T}$ and $y_0(t) = \frac{bt}{T}$ (T=exposure time), show that the degradation function becomes

$$H(u,v) = \frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-\pi i(ua+vb)}.$$

[3] Parametric Wiener Filter

(a) Implement a motion blurring filter as in problem [2] above.

(b) Blur image 5.26(a) in the +45o direction using $T = 1$, as in Fig. 5.26(b) ($a = b = 0.1$).

(c) Add a small amount of Gaussian noise of 0 mean to the blurred image. For this, you can use the Matlab command:

J = imnoise(I,'gaussian',0,v) adds Gaussian white noise of mean 0 and variance v to the image I (the parameter v needs to be specified). The default is zero mean noise with 0.01 variance.

(d) Restore the image using the parametric Wiener filter given by

$$\hat{F}(u,v) = \Big[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\Big]G(u,v),$$

where K is a specified constant, chosen to obtain best visual results.

(note that you may have to avoid division by zero in the formula above).

[4] (a) Recall the definition of the convolution $f * g(x,y)$ in continuous variables and two dimensions.

(b) Show that $\nabla^2(f * g) = f * (\nabla^2 g)$, where ∇^2 denotes the Laplace operator in (x,y).