Math 155, Vese

Reminder: Midterm exam, Monday, May 15, time 1-1.50pm (closed-note and closed-book exam, no calculators will be allowed).

Sections covered for the midterm: 2.3.4, 2.4 (except 2.4.4), 3.1, 3.2, 3.3, 3.4.1, 3.4.2, 3.5, 3.6, 3.7, Chapter 4 (except 4.3.4, 4.6.6, 4.6.7).

Definitions and proofs of properties of various filters and transforms will be given (up to 5 questions). There will be no programming questions.

Old midterm exam (Math 155, Spring 2004) will be posted on the class webpage.

Homework # 5, due on Wednesday, May 10

[1] (a) Implement the Gaussian lowpass filter in Eq. (4.3-8), using a radius $D_0 = 25$, and apply the algorithm to Fig4.11(a).

(b) Highpass the input image used in (a), using a highpass Gaussian filter of radius $D_0 = 25$ (see eq. (4.4-4)).

[2] Recall that the 1D DFT is

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-2\pi iux/M}.$$

Show the identity

$$f(x) = \sum_{u=0}^{M-1} F(u)e^{2\pi iux/M},$$

using the following orthogonality of exponentials

$$\sum_{u=0}^{M-1} e^{-2\pi iuy/M}e^{2\pi iux/M} = \begin{cases} M & \text{if } x = y \\ 0 & \text{otherwise.} \end{cases}$$

[3] (a) Show that $\mathcal{F}\Big(f(x,y)e^{2\pi i(u_0x/M+v_0y/N)}\Big) = F(u-u_0, v-v_0)$, where $F = \mathcal{F}(f)$.

(b) Using (a), deduce the formula used in shifting the center of the transform when $u_0 = M/2$ and $v_0 = N/2$, with M and N even positive integers.

[4] (a) Recall in two dimensions the Inverse Fourier Transform formula in continous variables (going from $F(u,v)$ to $f(x,y)$).

(b) Using (a), express the mixed partial derivative $\frac{\partial^2 f(x,y)}{\partial x \partial y}$ function of F.

[5] Consider the linear difference operator $g(x,y) = f(x+1,y) - f(x,y)$. Obtain the filter transfer function, $H(u,v)$, for performing the equivalent process in the frequency domain (Hint: use the identity (4.6-2) from page 195).