Math 155, Vese

Homework # 4 Due on Wednesday, May 3rd

[1] (a) Show using Taylor's expansion that the finite differences expression $\frac{f(x+h)-f(x-h)}{2h}$ is a second order approximation of the first-order derivative $f'(x)$.

(b) Apply this formula to approximate the gradient map

$$g(x,y) = |\nabla f|^2(x,y) = \Big(\frac{\partial f(x,y)}{\partial x}\Big)^2 + \Big(\frac{\partial f(x,y)}{\partial y}\Big)^2.$$

(c) Download Fig5.26a and plot an image negative of its gradient map g (edges will appear black, while homogenous regions will appear white). Explain the steps taken. Ignore the pixels on the boundary of the image for simplicity, when taking the discrete gradient.

[2] Show that the continuous 2D Fourier transform is a linear process.

[3] Compute in continuous variables the Fourier transform of the function

$$f(x) = \begin{cases} A, & \text{if } 0 \le x \le K, \\ 0, & \text{otherwise,} \end{cases}$$

where A and K are positive constants. Evaluate $F(0)$.

[4] Consider again the 2D continous Fourier transform and its inverse (denote by $H(u,v)$ the 2D Fourier transform of the spatial filter $h(x,y)$). Show that if the transform $H(u,v)$ is real and symmetric, i.e. if

$$H(u,v) = \overline{H(u,v)} = \overline{H(-u,-v)} = H(-u,-v),$$

then the corresponding spatial domain filter $h(x,y)$ is also real and symmetric.

Hint: need to use change of variables formula (here, $H(u,v)$ corresponds to a frequency domain filter).

[5] Fourier Spectrum and Average Value

(a) Download Fig5.26a and compute its (centered) Fourier spectrum.

(b) Display the spectrum.

(c) Use your result in (a) to compute the average value of the image.