Math 151A
HW #1, due on Wednesday, July 1st

[1] Using four-digit rounding arithmetic and rationalizing the numerator, find the most accurate approximations to the roots of the following quadratic equation. Compute the absolute errors and the relative errors.

\[
\frac{1}{3}x^2 - \frac{123}{4}x + \frac{1}{6} = 0.
\]

[2]
(a) Show that the polynomial nesting technique described in Section 1.2 can also be applied to the evaluation of

\[
f(x) = 1.01e^{4x} - 4.62e^{3x} - 3.11e^{2x} + 12.2e^x - 1.99.
\]

(b) Use three-digit rounding arithmetic, the assumption that \(e^{1.53} = 4.62\), and the fact that \(e^{nx} = (e^x)^n\) to evaluate \(f(1.53)\) as given in part (a).

(c) Redo the calculation in part (b) by first nesting the calculations.

(d) Compare the approximations in parts (b) and (c) to the true three-digit result \(f(1.53) = -7.61\).

[3]
(a) Show that the sequence \(p_n = (\frac{1}{10})^n\) converges linearly to \(p = 0\).

(b) Show that the sequence \(p_n = 10^{-2^n}\) converges quadratically to \(p = 0\).

You can use a hand calculator or the Bisection Algorithm posted on the class webpage for the following problems.

[4] Use the Bisection method to find \(p_3\) for \(f(x) = \sqrt{x} - \cos x\) on \([0,1]\).

[5] Find an approximation to \(\sqrt{3}\) correct to within \(10^{-4}\) using the Bisection Algorithm (hint: consider \(f(x) = x^2 - 3\)).

[6] Find a bound for the number of iterations needed to achieve an approximation with accuracy \(10^{-4}\) to the solution of \(x^3 - x - 1 = 0\) lying in the interval \([1, 2]\). Find an approximation to the root with this degree of accuracy.