Winter 2026 Math 131A
Homework Assignment 6

Problem 1. Ross 11.2(b)(c), 11.11

Problem 2. Ross 14.2(a)(b), 14.4(b)(c), 14.6(a)

Problem 3. Let (s_n) be a sequence and S be the set of all subsequential limits of (s_n). Prove that $\limsup s_n \in S$ and $\liminf s_n \in S$.

Problem 4. Suppose $(s_n)_{n\in\mathbb{N}}$ is a sequence such that the two subsequences $(s_{2k-1})_{k\in\mathbb{N}}$ and $(s_{2k})_{k\in\mathbb{N}}$ converge to the same limit $s \in \mathbb{R}$. Prove that $(s_n)_{n\in\mathbb{N}} \to s$.

Problem 5. Let $(s_n)_{n\in\mathbb{N}}$ be a Cauchy sequence. Suppose there exists some subsequence $(s_{n_k})_{k\in\mathbb{N}}$ such that $(s_{n_k})_{k\in\mathbb{N}} \to s \in \mathbb{R}$. Prove that $(s_n)_{n\in\mathbb{N}} \to s$.

Problem 6. Determine the convergence or divergence of the following series.

(i) $\sum_{n=1}^{\infty} \frac{1}{n}(\sqrt{n+1} - \sqrt{n})$

(ii) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(iii) $\sum_{n=1}^{\infty} \frac{1}{(\ln n)^n}$