Winter 2026 Math 131A
Homework Assignment 5

Problem 1. Ross 9.18

Problem 2. Ross 10.6, 10.7

Problem 3. Let (s_n) be a sequence of real numbers. Define $v_N = \sup\{s_n : n > N\}$.

(a) Prove that (v_N) is decreasing.

(b) Prove that if $v_M = +\infty$ for some $M \in \mathbb{N}$, then $v_N = +\infty$ for all $N \in \mathbb{N}$.

(c) Prove that if $\limsup s_n$ exists as a real number, then the sequence (s_n) is bounded from above and (s_n) has a convergent subsequence.

Problem 4. Let (s_n) be a sequence where $s_1 \neq s_2$. Suppose that there exists a real number $r \in (0, 1)$ such that

$$|s_{n+2} - s_{n+1}| \leq r|s_{n+1} - s_n|, \quad \forall n \in \mathbb{N}.$$

(a) Prove that

$$|s_{n+1} - s_n| \leq r^{n-1}|s_2 - s_1|, \quad \forall n \in \mathbb{N}.$$

(b) Prove that $(s_n)_{n\in\mathbb{N}}$ is a Cauchy sequence.

Problem 5. (a) Let (s_n) and (t_n) be two bounded sequences. Prove that

$$\liminf(s_n + t_n) \geq \liminf s_n + \liminf t_n.$$

(b) Give an example of two bounded sequences (s_n) and (t_n) such that

$$\liminf(s_n + t_n) > \liminf s_n + \liminf t_n.$$

Problem 6. Let (s_n) be a sequence satisfying

$$|s_i - s_j| \leq \frac{1}{ij} \quad \text{for all } i, j \in \mathbb{N}.$$

(a) Prove that $(s_n)_{n\in\mathbb{N}}$ is a Cauchy sequence.

(b) Prove that $(s_n)_{n\in\mathbb{N}}$ is a constant sequence, i.e., $s_i = s_j$ for all $i, j \in \mathbb{N}$.