Winter 2026 Math 131A
Homework Assignment 4

Problem 1. Ross 8.7(a), 8.9(a), 8.10.

Problem 2. Ross 9.4, 9.5, 9.10(a)(b).

Problem 3. Let $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ be two sequences. Suppose that $(a_n)_{n\in\mathbb{N}} \to a$ and $(b_n)_{n\in\mathbb{N}} \to b$ for some $a, b \in \mathbb{R}$. Prove that if $a_n \leq b_n$ for all $n \in \mathbb{N}$, then $a \leq b$.

Problem 4. Consider the sequence $(a_n)_{n\in\mathbb{N}}$ defined by $a_1 = 1$ and

$$a_{n+1} = \sqrt{a_n + 2}, \quad \text{for all } n \in \mathbb{N}.$$

(a) Prove that $0 \leq a_n < 2$ for all $n \in \mathbb{N}$.

(b) Show that the sequence $(a_n)_{n\in\mathbb{N}}$ is an increasing sequence.

(c) Show that $(a_n)_{n\in\mathbb{N}}$ is convergent and $(a_n)_{n\in\mathbb{N}} \to 2$.

Problem 5. Let $a, b \in \mathbb{R}$ such that $0 < a < b$. Define two sequences $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ such that $a_1 = a$, $b_1 = b$,

$$a_{n+1} = \sqrt{a_n b_n} \quad \text{and} \quad b_{n+1} = \frac{a_n + b_n}{2} \quad \text{for all } n \geq 1.$$

Prove that $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ are two convergent sequences converging to the same limit. *(Hint: use induction and the inequality $s + t \geq 2\sqrt{st}$ for any $s, t \geq 0$.)*