Winter 2026 Math 131A
Homework Assignment 3

Problem 1. Ross 7.3 (a)(c)(e), 7.4.

Problem 2. Ross 8.6, 8.8(b), 8.10.

Problem 3. Consider the sequence $(x_n)_{n\in\mathbb{N}}$ defined by $x_1 = \frac{1}{6}$ and

$$x_{n+1} = \frac{n+1}{n+3}\left(x_n + \frac{1}{2}\right) \text{ for any } n \geq 1.$$

Find x_{2026}. *(Hint: compute the first few terms and then use induction)*

Problem 4. In this exercise, we aim to prove that if a sequence converges, then its limit is unique. Let $(a_n)_{n\in\mathbb{N}}$ be a convergent sequence that has a limit a, i.e.,

$$\textit{for every } \epsilon > 0 \textit{ there exists a number } N \textit{ such that if } n \geq N \textit{ then } |a_n - a| < \epsilon. \qquad (*)$$

Show that if the sequence has another limit $\tilde{a}$ satisfying (*), then $\tilde{a} = a$.