UCLA Department of Mathematics

Math 132 Section 2 Spring 2021

Name: ______ ______ ______
Last First MI

Section:

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Final Problem Set

(Massive)

Problem (1) Let $f(z) = \frac{1}{(z-a)^3} \tanh z$. Using the derivative formulas, compute $\mathrm{Res}_f(a)$.

Problem (2) For the usual (branch) choice, namely $-\pi < \theta \leq \pi$ let $f(z) = \frac{z^{1/2}}{(z-a)^3}$ with a real and positive. Compute $\mathrm{Res}_f(a)$.

Problem (3) Use the derivative formula (or other methods) to compute the residue of $\frac{\cos z}{(z-ia)^m}$ where m is a positive integer and a is a real number.

Problem (4) Let $f(z)$ denote the function

$$f(z) = \frac{e^{\frac{1}{z}}}{1-z}.$$

Compute $\oint_\gamma f(z)dz$ where γ is any contour that encloses the origin but does not enclose the point $z = 1$.

Problem (5) On the basis of simple residue theory, compute $\oint_\gamma \frac{dz}{z\sin z}$ where γ is any "small" circle – radius less than π that encircles the origin.

Problem (6) Using either the derivative formula – many times, or via a hands on approach, compute

$$\int_0^{2\pi} \mathrm{e}^{2\cos\theta} d\theta.$$

Remark: There is no easy way around this one; your answer should be in the form of an infinite sum which, as it turns out, is well tabulated.

Problem (7) Using the method of residues, evaluate

$$\oint_{|z|=8} \frac{dz}{z^2+z+1}$$

and check your answer by an alternative (but contour based) method.

Problem (8) Evaluate, via the residue theorem, the following integral on the positive circular contour centered at the origin:

$$\oint_{|z|=3} \frac{\mathrm{e}^z}{z(z-2)^2} dz$$

Problem (9) Evaluate, via the residue theorem, the following integral on the positive circular contour centered at the origin:

$$\oint_{|z|=1} \frac{dz}{z^2 \sin z}.$$

Problem (10) Evaluate

$$\int_0^{2\pi} \frac{d\theta}{1+\sin^2\theta}.$$

Problem (11) Evaluate

$$\int_0^{2\pi} \frac{\sin^2\theta d\theta}{a+b\cos\theta}$$

where a and b are real numbers that satisfy $a > b > 0$.

Problem (12) Evaluate

$$\int_0^{2\pi} \frac{d\theta}{[a+\sin^2\theta]^2}$$

where a is real and $|a| < 1$.

Problem (13) Evaluate, for integer n and complex β with $|\beta| < 1$,

$$\int_0^{2\pi} \frac{d\omega}{1 + \beta \cos n\omega}.$$

Think hard about this one before using methods of complex analysis.

Problem (14) Evaluate, using the method of residues,

$$\int_{-\infty}^{+\infty} \frac{1}{1 + x^4} dx.$$

Problem (15) Evaluate, using the method of residues,

$$\int_{-\infty}^{+\infty} \frac{1+x^2}{1+x^4} dx.$$

Problem (16) Evaluate, using the method of residues,

$$\int_{-\infty}^{+\infty} \frac{x^2 dx}{(1+x^2)^3}.$$

Problem (17) Evaluate, using the method of residues,

$$\int_{-\infty}^{+\infty} \frac{\cos x dx}{(1+x^2)^2}.$$

Problem (18) Evaluate, using the method of residues,

$$\int_{-\infty}^{+\infty} \frac{\sin x \cos x}{x^2+2x+2} dx.$$

Problem (19) ($\star$) Evaluate, using the method of residues,

$$\int_{-\infty}^{+\infty} \frac{x \sin x}{x^2 - 2x + 10}.$$

Justify, a little, your method.

Problem (20) ($\star$) Evaluate, using the method of residues and a certain amount of cleverness, the integral

$$\int_0^{\infty} \frac{dx}{1 + x^3}.$$