Name: ______________________________ Student ID: ____________ Section: 2___

Prof. Alan J. Laub May 1, 2009

Math 33A – MIDTERM EXAMINATION
Spring 2009

Instructions:

(a) The exam is closed-book (except for one page of notes) and will last 50 minutes.

(b) Notation will conform as closely as possible to the standard notation used in the lectures, not the textbook.

(c) Do all 4 problems. Each problem is worth 25 points. Hand in the exam with work shown where appropriate, especially on problems 1, 2, and 4. Some partial credit may be assigned if warranted.

(d) Label clearly the problem number and the material you wish to be graded.

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1. Consider the vector space $\mathbb{R}^{n\times n}$ over $\mathbb{R}$ and let $\mathcal{S}$ be the subset of symmetric $n \times n$ matrices (i.e., matrices A for which $A^T = A$) and let $\mathcal{K}$ be the subset of skew-symmetric matrices (i.e., matrices A for which $A^T = -A$).

   (a) Show that $\mathcal{S} \subseteq \mathbb{R}^{n\times n}$ and $\mathcal{K} \subseteq \mathbb{R}^{n\times n}$.

   (b) Show that $\mathcal{S} \oplus \mathcal{K} = \mathbb{R}^{n\times n}$. You will find the matrix identity

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

   useful.

2. Let $\mathcal{S} = \mathrm{Sp}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$. Find a basis for $\mathcal{S}^\perp$.

3. Suppose the matrix $A \in \mathbb{R}_n^{n\times n}$ is known to have an LU factorization, i.e., $A = LU$. What then are the block LU factorizations, in terms of L and U, of the following $2n \times 2n$ block matrices?

   (a) $\begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}$

   (b) $\begin{bmatrix} A & A \\ 0 & A \end{bmatrix}$

   (c) $\begin{bmatrix} A & 0 \\ A & A \end{bmatrix}$

   (d) $\begin{bmatrix} A & A \\ A & 0 \end{bmatrix}$

   (e) $\begin{bmatrix} A & A \\ A & 2A \end{bmatrix}$

4. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 5 & 5 & 3 \end{bmatrix} \in \mathbb{R}^{3\times 4}.$$

(a) Put A in row-reduced echelon form (rref).

(b) What is $\operatorname{rank}(A)$? What is $\operatorname{rank}(A^T)$?

(c) Find a basis for $\mathcal{N}(A)$ from $\operatorname{rref}(A)$.

(d) Find a basis for $\mathcal{C}(A)$ from $\operatorname{rref}(A)$.

(e) Find a basis for $\mathcal{C}(A^T)$ from $\operatorname{rref}(A)$.

(f) The matrix $E = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -3 & 1 \end{bmatrix}$ puts A in rref. Using E, find a basis for $\mathcal{N}(A^T)$.