32B Killip Homework 7 Due March 2nd

(1) For which functions $g(y,z)$ is the vector field

$$\vec{F}(x,y,z) = \begin{bmatrix} g(y,z) \\ y+xz \\ xy \end{bmatrix}$$

conservative? Find all potential functions in all cases.

(2) The blade of a fan is described in cylindrical polar coordinates by the equations

$$1 \leq r \leq 2 \qquad -\pi/12 < \theta < \pi/12 \qquad z = \theta$$

Write this as a parametric surface and so compute its area.

(3) Approximating the earth by a sphere, find the latitude above which lies one quarter of the earth's surface. (Verify that Los Angeles lies in this 'northern quarter' of the earth.)

(4) Book Exercises 17.4: 4, 28, 34, 42.

(5) Book Exercises 17.5: 22, 26, 30, 36.

Some additional practice problems appear on the next page.

- Let D denote the region where $0 \leq x \leq y \leq 1$. Compute
$$\iint_D \frac{x}{\sqrt{1-y^2}}\, dA$$
by making the change of variables $x = v\sin(u)$, $y = \sin(u)$.

- We wish to build a fence on the landscape
$$z = -1 - 8x^2 - 16y^2$$
from ground level up to height $z = 0$. The path of the fence is $y = x^2$ as x varies from $x = -2$ to $x = +2$. What will be the area of (the face of) the fence. Leave your answer as a one-variable integral.

- How much work is done by a fish swimming in a straight line from $(0, 0)$ to $(3, 7)$ against the force of the current
$$\vec{F} = \begin{bmatrix} 3 - x^3 + y \\ y^3 \end{bmatrix}$$

- Consider the 'vortex field'
$$\vec{F} = \begin{bmatrix} \frac{-y}{x^2+y^2} \\ \frac{x}{x^2+y^2} \end{bmatrix}$$
as defined in Section 17.3 of the text. As the book explains, it is not conservative if considered as a vector field on $\mathcal{D} = \{(x, y) \neq (0, 0)\}$. Argue by Theorem 4 that $\vec{F}$ is conservative if considered as a vector field on the quadrant where $x > 0$ and $y > 0$. Further verify this by giving a potential function valid in this quadrant.

- The idealized magnetic dipole in 3D is given by
$$\vec{B} = \nabla \times \vec{A} \quad \text{with} \quad \vec{A} = (x^2 + y^2 + z^2)^{-3/2} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
(a) Compute $\vec{B}$ explicitly. Where is it defined?
(b) Is this region simply connected?
(c) Is $\vec{A}$ conservative? Is $\vec{B}$ conservative?
(d) If either is conservative, find its potential function.