32A Killip Homework 8 Due Friday, Dec. 8.

- (a) Find all critical points of the function
  $$f(x,y,z) = (x+4y+z)\exp\left(\tfrac{1}{2} - x^2 - y^2 - z^2\right)$$
  (b) Pick one and determine the approximating paraboloid at that point.
  (c) Complete squares to determine if the critical point is a maximum, a minimum, or a saddle.
- Section 15.7: 35, 44, 54.
- Consider three fixed points $\vec{x}_1$, $\vec{x}_2$, and $\vec{x}_3$ in space. Find the point $\vec{x}$ that minimizes the sum of distances squared, that is, which minimizes
  $$f(\vec{x}) = \|\vec{x} - \vec{x}_1\|^2 + \|\vec{x} - \vec{x}_2\|^2 + \|\vec{x} - \vec{x}_3\|^2$$
  Incidentally, if $\vec{x}$ was tied to the three points via (idealized) elastic bands, this minimum would be the equilibrium position (i.e. the position of least energy).
  Hint: Use vectors as much as possible — writing out all the components gets very messy.
- Section 15.8: 2, 10, 22, 52.
- Fix a unit vector $\vec{n}$ as well as two points $\vec{x}_1$ and $\vec{x}_2$ in the plane.
  (a) Show that any point $\vec{x}$ on the line
  $$\vec{n} \cdot \vec{x} = 0$$
  that minimizes
  $$f(\vec{x}) = \|\vec{x} - \vec{x}_1\| + \|\vec{x} - \vec{x}_2\|$$
  must have the property
  $$\left(\frac{\vec{x} - \vec{x}_1}{\|\vec{x} - \vec{x}_1\|}\right)_{\perp \vec{n}} + \left(\frac{\vec{x} - \vec{x}_2}{\|\vec{x} - \vec{x}_2\|}\right)_{\perp \vec{n}} = 0.$$
  We assume here that neither $\vec{x}_1$ nor $\vec{x}_2$ lie on the line $\vec{n} \cdot \vec{x} = 0$.
  (b) Observe that the previous relation can be expressed as follows: "the angle of incidence is equal to the angle of reflection", at least if both points are on the same side of the 'mirror' occupying the line $\vec{n} \cdot \vec{x} = 0$.