Consider the surface $z^2 = 1 + x^2 + y^2$.

(a) Find all points on the surface where the tangent plane is perpendicular to $$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

(b) Find all points on the surface whose tangent plane passes through $(0,0,1/2)$.

• Section 15.6: 4, 13, 20, 21, 30, 44, 46.

• Compute all partial derivatives of
  \[ F(x, y, z) = \int_x^y e^{s^2} - \frac{1}{s} \, ds \]
  and so determine
  \[ \frac{d}{dt} \int_t^{2t} e^{s^2} - \frac{1}{s} \, ds \]

• Section 15.7: 4, 12, 24, 26.

• In class, we introduced the paraboloid of best approximation to $f(x, y)$ at $(0,0)$ as
  \[ p(x, y) = f(0,0) + \nabla f(0,0) \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} (0,0) x^2 + \frac{\partial^2 f}{\partial x \partial y} (0,0) xy + \frac{1}{2} \frac{\partial^2 f}{\partial y^2} (0,0) y^2 \]
  Show that for any vector $\vec{u} = (a, b)$ we have
  \[ \frac{d}{dt} \bigg|_{t=0} f(t\vec{u}) = \frac{d}{dt} \bigg|_{t=0} p(t\vec{u}) \]
  and
  \[ \frac{d^2}{dt^2} \bigg|_{t=0} f(t\vec{u}) = \frac{d^2}{dt^2} \bigg|_{t=0} p(t\vec{u}) \]

*Hint:* Problem 15.6.46, assigned earlier, helps with the latter.