32A Killip Homework 4 Due Friday, Oct. 27

- Fix $0 < b \leq a$ and consider the parametric curve
$$\vec{r}(t) = \begin{pmatrix} a\cos(t) \\ b\sin(t) \end{pmatrix},$$
which is an ellipse.
(a) Find the unit tangents $\vec{T}(t)$ and normals $\vec{N}(t)$ to the ellipse.
(b) Find the curvature of the ellipse as a function of t (cf. Exercise 14.4.25).
(c) Find the points of minimal and maximal curvature (if $a > b$) and mark them on a sketch.
(d) Find the osculating circles at the points of maximal and minimal curvature and mark them on your sketch. Include explicitly the locations of the centers.
(e) Defining $c = \sqrt{a^2 - b^2}$, show that the perimeter of the triangle with vertices
$$\vec{r}(t), \quad F_1 = \begin{pmatrix} c \\ 0 \end{pmatrix}, \quad \text{and} \quad F_2 = \begin{pmatrix} -c \\ 0 \end{pmatrix}$$
is independent of t and determine its value. The points F_1 and F_2 are called the foci of the ellipse.
(f) Show that the two vectors pointing to $\vec{r}(t)$ from each of the foci make equal angles with $\vec{N}(t)$.
- From Section 14.4: 34, 69, 82.
- From Section 14.5: 4, 49 (but give you answer in meters per second squared).
- From Section 14.6: 17.
- Chapter 14 review exercises: 26, 40.