(1) Consider the following processes: There are $N_0 = 1$ many individuals in the zeroth generation. The number of individuals N_k in the k th generation comes from each individual in the $(k-1)$ th generation having Poisson(λ) many offspring independent of all others.
(a) Find a formula for $\mathbb{E}(N_k)$.
(b) Suppose $\lambda < 1$. Show that $\mathbb{P}(N_k = 0)$ converges to unity as $k \to \infty$.

(2) Consider the processes from the previous problem modified so that the number of offspring which each individual has is Bernoulli(p) distributed.
(a) Use moment generating functions to determine the law of N_k for each k.
(b) Use the above to determine $\mathbb{P}(N_k = 0)$.
(c) Give a simpler direct determination of $\mathbb{P}(N_k = 0)$.

(3) Let X be a random variable for which $M_X(s)$ is finite for all $s \in \mathbb{R}$.
(a) Show that $\ln M_X(s)$ is a convex function of s.
(b) Deduce that $\ln M_x(s) \geq s\mathbb{E}(X)$.

(4) Fix $p \in (0,1)$, let $b \geq p$, and let $X \sim$ Binomial(n, p). In class, we showed that

$$\mathbb{P}(X \geq an) \leq e^{-nH} \quad \text{with} \quad H = a\log\left(\tfrac{a}{p}\right) + (1-a)\log\left(\tfrac{1-a}{1-p}\right)$$

by following the Cramér/Chernoff method; see also Problem 5.2.
(a) Find an analogous bound for $\mathbb{P}(X \leq an)$ for $a < p$. [Mimic the above but with $s \leq 0$.]
(b) A candidate claims 75% support for their policy. If this is true, give an upper bound on the probability that a sample of ten thousand people shows less than 50% support.