170B Killip Midterm 2 February 18th

First Name: ____________________ ID#____________________

Last Name: ____________________

Rules.

- There are **FOUR** problems; fifteen points per problem.
- There are extra pages after each problem. You may also use the backs of pages.
- No calculators, computers, notes, books, crib-sheets,...
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring,... Try to sit still.
- Turn off your cell-phone, pager,...

1	2	3	4	$\sum$

(1) (a) State the Strong Law of Large Numbers.
(b) State the Weak Law of Large Numbers.
(c) Define what it means for a sequence of random variables X_n to converge to a random variable X *in distribution*.
(d) State the Markov inequality.

$\boxed{\text{extra paper}}$

(2) Recall that a random variable X has law Exponential(λ) if the pdf of X is given by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & : x \geq 0 \\ 0 & : x < 0. \end{cases}$$

(a) Find the moment generating function of $X \sim$ Exponential(λ).
(b) Use the Cramér/Chernoff method to give an upper bound on $\mathbb{P}(X \geq \frac{2}{\lambda})$.

$\boxed{\text{extra paper}}$

(3) Recall that we say $X \sim \text{Geometric}(p)$ if

$$\mathbb{P}(X = k) = p(1-p)^{k-1} \qquad \text{for each } k = 1, 2, 3, \ldots.$$

For such a random variable, the moment generating function is

$$M_X(s) = \frac{p}{e^{-s} - (1-p)}$$

Let $N \sim \text{Geometric}(p)$. And let Y denote the sum of N independent identically distributed random variables $X_i \sim$Geometric(p).
(a) What are

$$\mathbb{E}(Y|N), \qquad \mathbb{E}(Y), \qquad \text{var}(Y|N), \qquad \text{var}(\mathbb{E}(Y|N)), \quad \text{and} \quad \text{var}(Y).$$

(b) What is the moment generating function of Y?

$\boxed{\text{extra paper}}$

(4) (a) State the Central Limit Theorem

(b) Use the central limit theorem to estimate the probability that 100 tosses of a fair coin results in exactly 50 heads. A table of Φ is available on the last page of the exam.

$\boxed{\text{extra paper}}$