

First Name: _____ ID# _____

Last Name: _____

Rules.

- There are **FOUR** problems; ten points per problem.
- There are extra pages after each problem. You may also use the backs of pages.
- No calculators, computers, notes, books, crib-sheets,...
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring, ...
Try to sit still.
- Turn off your cell-phone, pager,...

1	2	3	4	Σ

- (1)
 - (a) Define the (Pearson) correlation coefficient of two random variables X and Y .
 - (b) State the Cauchy–Schwarz inequality.
 - (c) State the law of iterated expectation.
 - (d) If $\text{var}(X) = 7$, $\text{var}(Y) = 3$, and $\text{cov}(X, Y) = 1$, what is $\text{var}(2Y - X)$?

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- (2) (a) Let Z be uniformly distributed on the interval $-1 \leq z \leq 1$. What is the pdf of Z^2 .
(b) Suppose the joint pdf of X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} 2 & : 0 \leq y \leq x \leq 1 \\ 0 & : \text{otherwise} \end{cases}$$

What is the pdf of $\frac{Y}{X}$?

For full credit, you should indicate the values of the pdf on all parts of the line.

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(3) Let Y denote the number of eggs laid by a turtle. It is Poisson distributed with $\mathbb{E}(Y) = \lambda$. The probability that an egg produces a turtle that survives to adulthood is p and independent for each egg. Let X denote the number of such eggs that survive to adulthood. Thus, conditioned on Y the law of X is Binomial(p, Y).

- (a) Determine $E(X|Y)$ and $E(X)$.
- (b) Determine $\text{var}(E(X|Y))$ and $\text{var}(X)$.
- (c) Determine the covariance of X and Y .

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- (4) Consider the sample space $\Omega = \{a, b, c\}$. The probability of each outcome is listed in the table below, as are the values of two random variables X and Y .

Complete the table by entering the values of the remaining random variables.

ω	$\mathbb{P}(\omega)$	X	Y	$\mathbb{E}(Y X)$	$\text{var}(Y X)$
a	$\frac{1}{2}$	0	2		
b	$\frac{1}{4}$	0	4		
c	$\frac{1}{4}$	1	8		

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