First Name: _____   ID#_____

Last Name: _____

Rules.

- There are **FIVE** problems, totaling 50 points.

- There are extra pages after some problems. You may also use the backs of pages.

- No calculators, computers, notes, books, crib-sheets,...

- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring,... Try to sit still.

- Turn off your cell-phone, pager,...

1	2	3	4	5	$\sum$
/7	/8	/10	/10	/15	/50

(1) Let X and Y be random variables with $\text{var}(X) = 12$ and $\text{var}(Y) = 3$.
    (a) According to the Cauchy–Schwarz inequality, what is the largest possible value of $\text{cov}(X, Y)$?

Suppose now that we discover $\text{cov}(X, Y) = 1$.
(b) What is $\rho(X, Y)$?
(c) What is $\text{var}(X - 2Y)$?

(2) Suppose X is uniformly distributed on the interval $[0, 1]$. Determine the pdf of $Y = \dfrac{1}{X}$.

(3) Let X denote the number of heads resulting from two (independent) tosses of a fair coin.
  (a) Sketch the CDF of X. Label your axes thoroughly.
  (b) Determine the MGF of $Y = (X - 1)^2$.
  (c) Find $\mathbb{E}(Y|X)$.

extra paper

(4) Let X and Y be random variables whose joint pdf is as follows:

$$f_{X,Y}(x,y) = \begin{cases} 2 & : \text{if } x \geq 0, \ y \geq 0, \ \text{and } x + y \leq 1 \\ 0 & : \text{otherwise} \end{cases}$$

(a) Find the pdf of $Z = X + Y$.

(b) What is the probability that $Z \leq \frac{2}{3}$ given that $X \geq \frac{1}{3}$.

extra paper

(5) Let X and Y be random variables with

$$f_X(x) = \begin{cases} 1 & : \ 0 < x \le 1 \\ 0 & : \ \text{otherwise} \end{cases} \quad \text{and} \quad f_{Y|X}(y|x) = \begin{cases} \frac{1}{x} & : \ 0 < y \le x \\ 0 & : \ \text{otherwise.} \end{cases}$$

(a) Determine $\mathbb{E}(Y|X)$.
(b) Determine $\mathbb{E}(Y)$.
(c) Determine $\text{var}(Y|X)$.
(d) Determine $\text{var}(Y)$.

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