MATH 1 REVIEW ON GRAPHING FUNCTIONS

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OUTLINE

I will write up a few examples and practice problems with an answer key in the back 4. All the examples are already in factored form, if you have a polynomial that you wish to sketch the first step is to put it in factored form. To keep these notes focused I will not cover how to factor polynomials or long division of polynomials. I include both examples and problems, the intension is that you can read and follow through examples and that you try the problems on your own (but there are solutions in the answer key).

Example 1

As a warm-up lets begin with an example of graphing a polynomial as this is the foundation which we will build on to graph rational functions. Sketch the following function, be sure to include all intercepts and end behavior:

$$f(x) = \frac{-1}{4}(x+2)^2(x^2+3)(2x-3) \tag{1}$$

Before we start graphing lets record some important details. The most important and typically easiest to find are the x-intercepts. In addition to finding them we want to record their multiplicity: even vs odd (or bounce vs cross). I will put a subscript b for even/bounce and a subscript c for odd/cross.

$$\text{x-int: } -2_b, 1.5_c$$

To find this we need to look at each factor and ask when it is zero. The first factor is $(x+2)$ which is zero when $x=-2$ and since this factor is squared (power 2) we know it is a bounce (to an even power is bounce). The next factor is (x^2+3) we know that this is never zero as $x^2 \geq 0$ and 3 is positive so there are no zeros here. (Note that (x^2-3) is a difference of squares and should be written as $(x-\sqrt{3})(x+\sqrt{3})$ to be in factored form). The last factor is $(2x-7)$ which is zero when $x=\frac{3}{2}=1.5$, we recall that no power means it is to the power 1 and this is odd so we label the zero as a cross.

Next up we look piece of information we should ask is end-behavior. For this we need to find the leading coefficient which is multiplying all the leading coefficients of each factor and the number out front so we get that this is $(\frac{-1}{4})\cdot(x)^2\cdot(x^2)\cdot(2x) = \frac{-1}{2}x^5$, note that as the $(x+2)^2$ is squared its leading coefficient is squared, if the factor had been instead $(3x+2)^2$ the term would be $(3x)^2 = 9x^2$. There are two things we care about this term, its sign and its degree, the degree is 5 which is odd so the end-behavior is either $-\infty$ to the left and $+\infty$ to the right or $-\infty$ to the right and $+\infty$ to the left. The sign tells us which one, since it is negative it is $-\infty$

Date: November 4, 2026.

to the right and $+\infty$ to the left (the easiest way to recall this is to plug in 1 and -1 to the leading coefficient and see where it is positive vs negative).

Finally, lets find the y-intercept, this involves just plugging in for x and we get

$$\text{y-int:} = \frac{-1}{4} \cdot (2)^2(3)(-3) = 9$$

Before I sketch, I draw all the important information they asked me to include (intercepts and end behavior) After this I fill in the details:

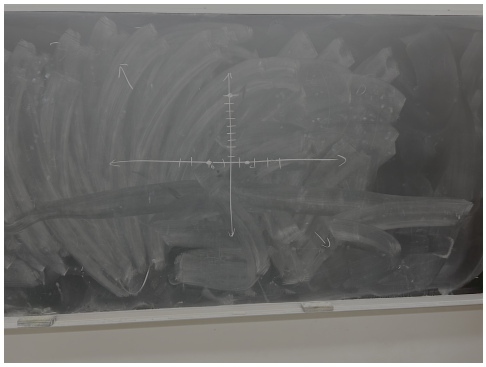
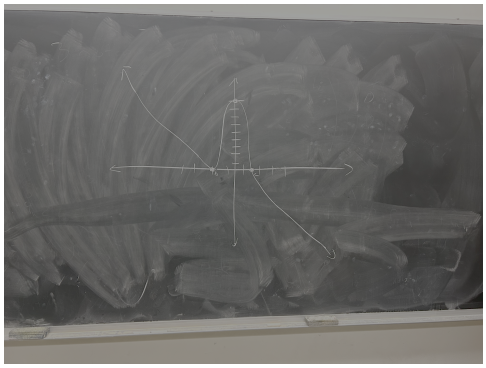
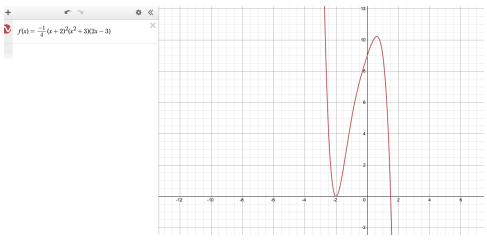


Figure 1: All the data with my graph note how I like to mark if the x intercepts are bounce or cross this is not necessary but I think it is helpful.



If I graph it on desmos I get Note my sketch is not perfect, I had a peak at $(0, 9)$



whereas the real peak was $(.6, 10)$ this is not what we as graders care about what

we want is general shape and to see where the function is greater than 0 and less than 0.

Problem for you

Now that you have this try one on your own sketch

$$g(x) = (2x-1)^4(x+1)^8(x^2+5) \tag{2}$$

include all intercepts and end behavior.

Example 2

Graph the following rational function include all intercepts, holes, and asymptotes.

$$h(x) = \frac{4(x-3)(2x-1)^2}{5(x-3)(x-4)^2} \tag{3}$$

Again, we are going to start with recording all the information. The first thing to check is if there are any holes, that is if there are any factors which are on top and on bottom, we see that there are there is an $(x-3)$ on top and bottom and so we cross them out and add a hole at $x=3$ we are left with the equation,

$$h(x) = \frac{4(2x-1)^2}{5(x-4)^2}$$

It has come to my attention that you do not need to compute the exact y-coordinate of the hole you can just plot it in the approximate position in your sketch [1] so

$$\text{holes:}(3,20)$$

Next, we find the asymptotes, for the vertical asymptotes we look for the zeros of the denominator (after removing the hole) and get $(x-4)^2$ which is zero when $x=4$ and has multiplicity 2 so it is even.

$$\text{VA:}4_e$$

At this point we have found the domain of the function $h(x)$, it is every value of x except that of the hole and the the VA, is in words it is every real number other than $x=3$ and $x=4$ which can be written in interval notation as $(-\infty,1)\cup(1,4)\cup(4,+\infty)$, the problem I wrote did not ask for this but it is good to be able to answer such a question if it does appear on a quiz or test.

For the horizontal asymptotes we look at the leading term of the top and bottom, then top has a leading term (after the hole was removed) of $4\cdot(2x)^2 = 8x^2$ and the denominator has a term of $5x^2$ these are the same power so the HA is at the faction they make which is $\frac{8}{5}$.

$$\text{HA:}\frac{8}{5}$$

Next, the intercepts. The x-intercepts are when the top is zero (after we remove the hole!) so the only fact is $(2x+1)^2$ which is zero when $x=-0.5$ and has multiplicity 2 which is even.

$$\text{x-int:}0.5_b$$

[1] To find the y-coordinate of the hole we need to plug in 3 and get $\frac{4(5)^2}{5\ (-1)^2} = \frac{4\cdot 5}{1} = 20$ (tip: cancel as much as possible before you divide).

For the y-intercepts we plug in 0 and are left with $\frac{4\cdot 1^2}{5(-4)^2} = \frac{1}{7\cdot 4} = \frac{1}{20} > 0$ (again we cancel then divide).

$$\text{y-int:} \frac{1}{20}$$

Again, we can sketch the data we just collected first,

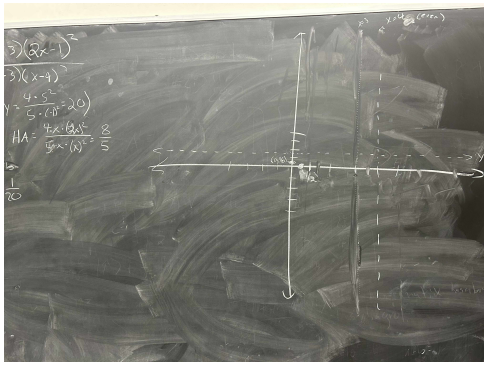


Figure 2: Marking all HA, VA, intercepts and holes

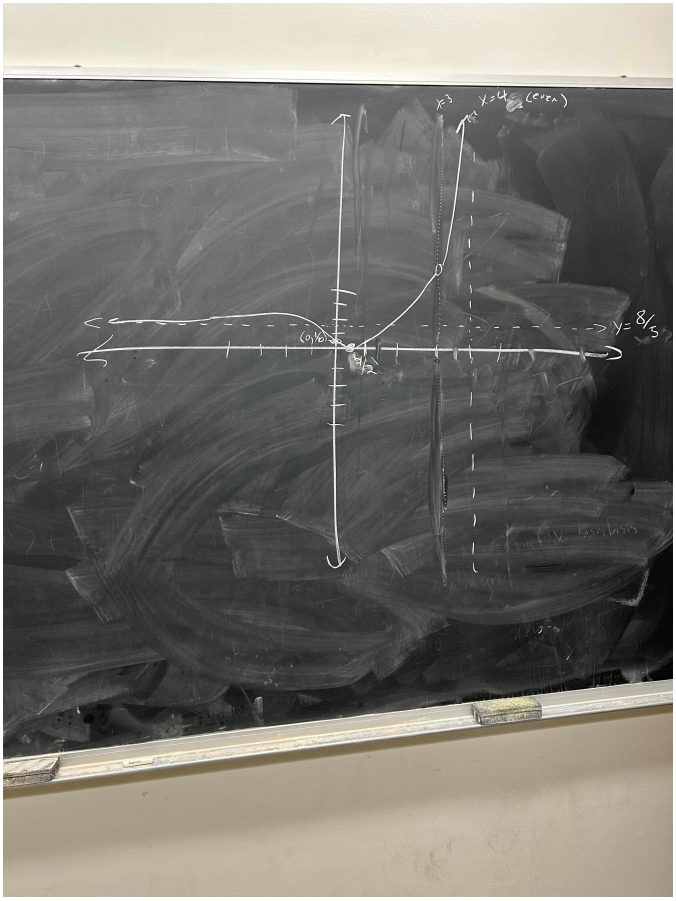


Figure 3: Doing the region containing the y-intercept

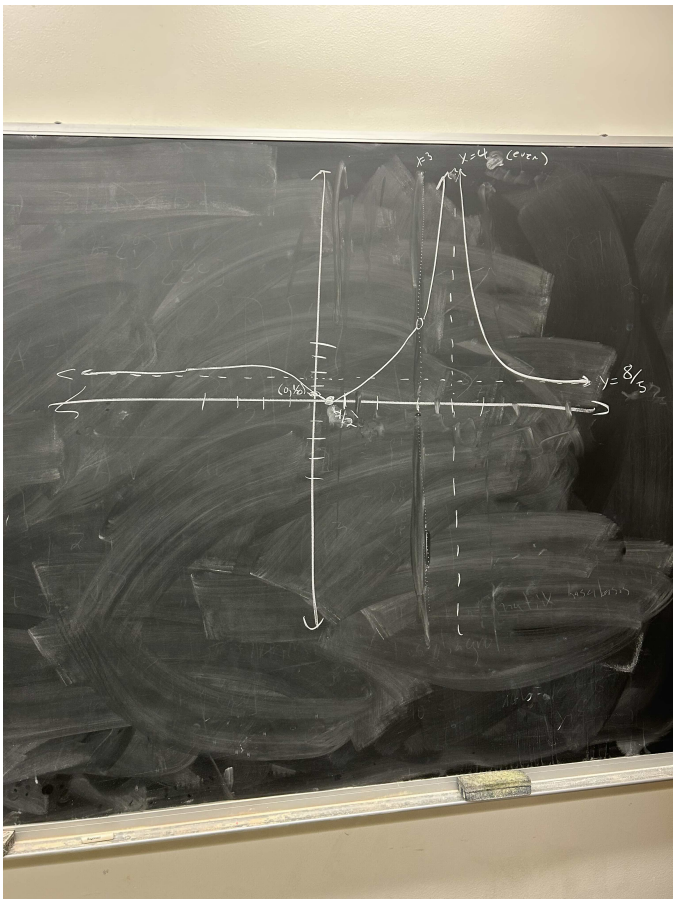


Figure 4: Because the VA at 4 is odd we know that one side of the graph will be at $+\infty$ and the other at $-\infty$, since the graph is negative just to the left it must be positive just to the right, we then continue to draw the rest of the right region. Alternatively we can test the point $x = 5$ and see that it give $h(5) = \frac{4\cdot(9)^2}{7\cdot 1^2} > 0$ and then sketch from that point.

Problems for you

Consider the following graphs they are supposed to be increasing in difficulty:

$$a(x) = \frac{12(x-2)}{(x+3)(x+2)^2(x-4)} \tag{4}$$

$$\frac{12(x+1)(x-1)^2}{(x+1)(x-2)(x+4)} \tag{5}$$

Answer Key

For the problem for equation 2 we have:

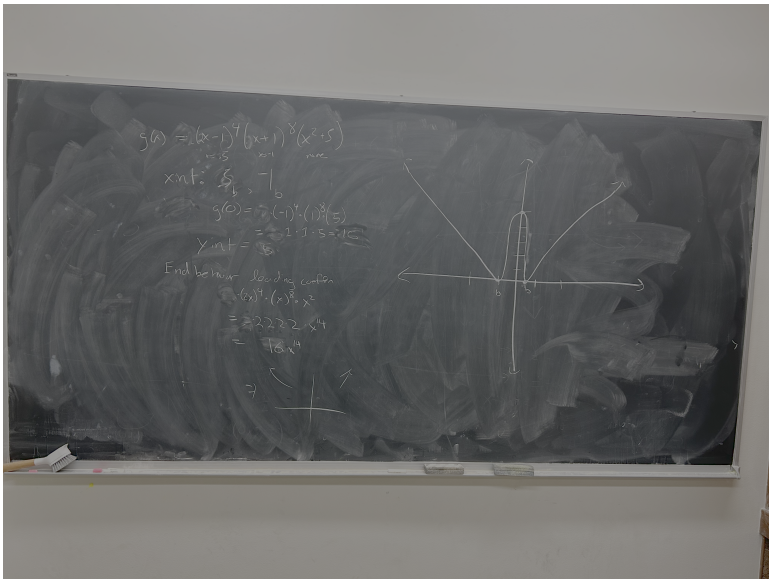
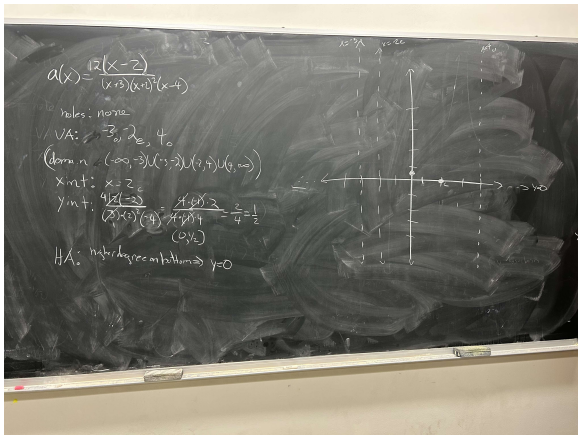


Figure 5: Solution for equation 2

For the problem for equation 4 we have the set up:



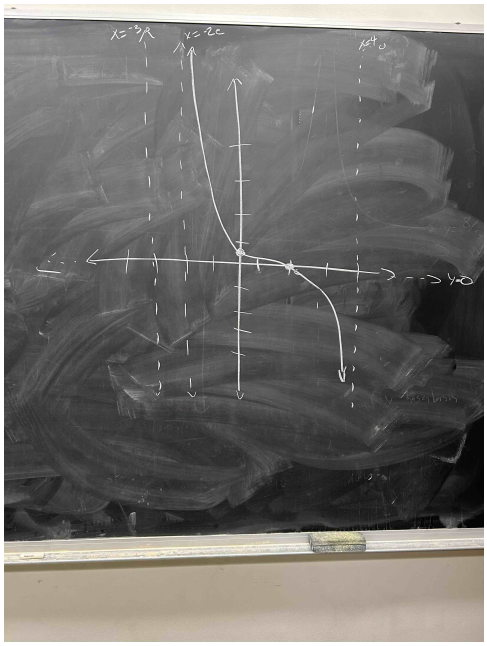


Figure 6: start with region containing y int

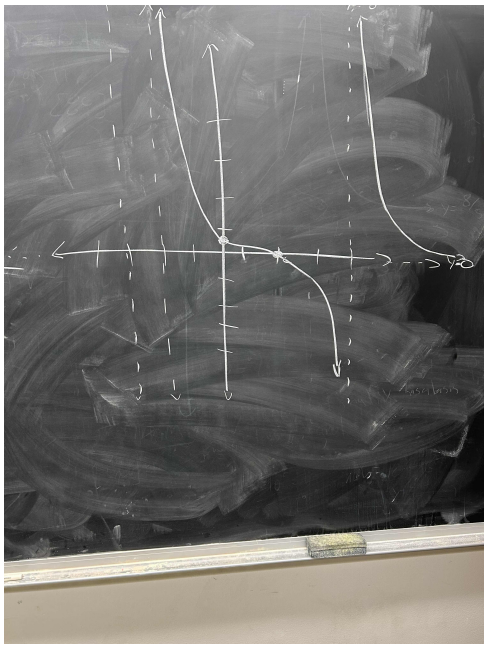


Figure 7: Either use that the asymptote is odd at $x = 4$ or plug in $a(5) = \frac{12(3)}{(8)(7)^2(1)} = \frac{9}{98} > 0$

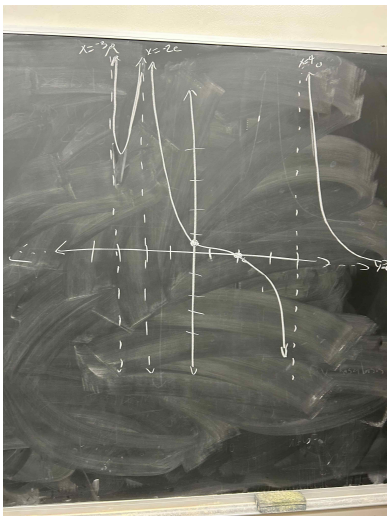


Figure 8: Either use that the asymptote is even at $x = -2$ or plug in $a(-2.5) = \frac{12(-4.5)}{(.5)(-.5)^2(-6.5)} > 0$. This problem was set up to use the even vs odd this fraction is likely more gnarly than the ones you will be tested on, but since you only need to tell if it is positive or negative you should be able to do this without a calculator.

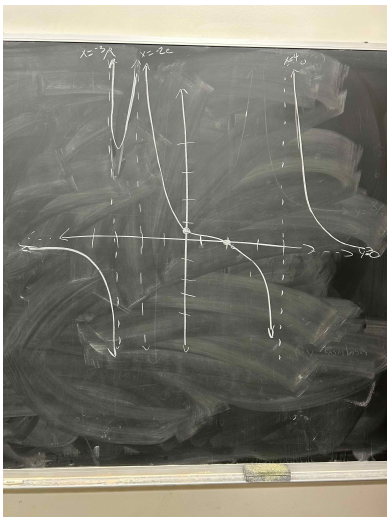


Figure 9: Either use that the asymptote is odd at $x = -3$ or plug in $a(-4) = \frac{12(-6)}{(-1)(-2)^2(-8)} = \frac{-9}{4} < 0$.

Solution for equation 5:

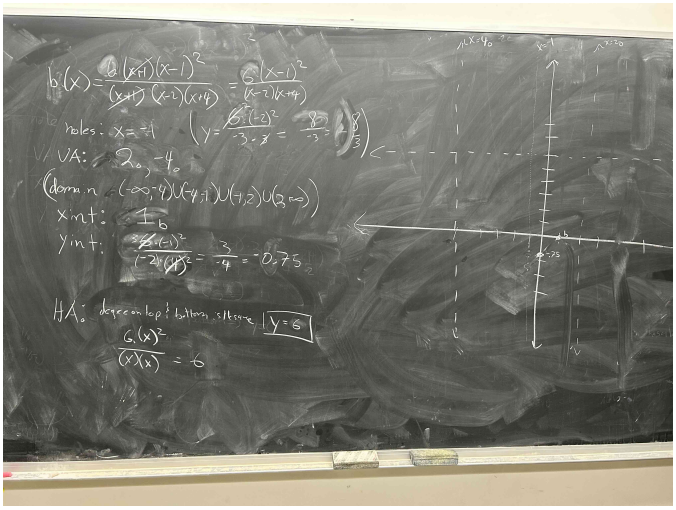


Figure 10: Set-up

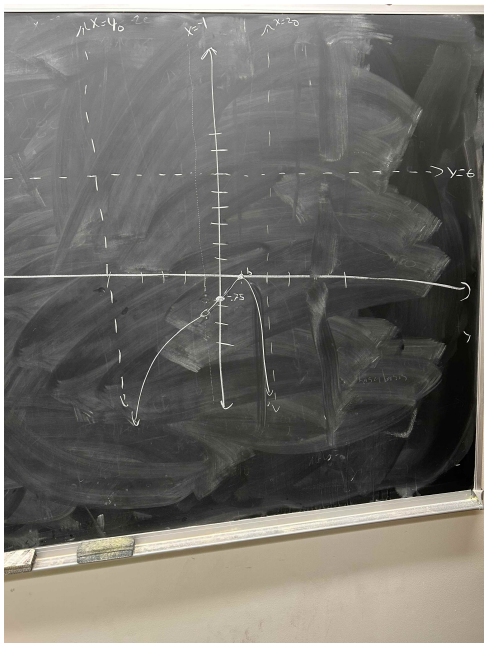


Figure 11: Start with region containing y-int

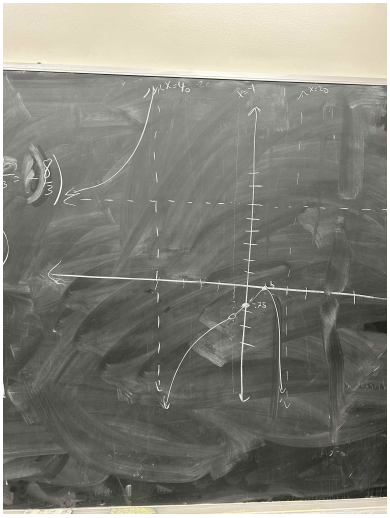


Figure 12: Use that the asymptote at $x = -4$ is odd or plug in $b(-5) = \frac{6(4)^2}{(-7)(-1)} > 0$ or since the HA is not zero start from the HA. If you are plugging in the numbers not all you care about is if it is positive are negative so the nasty numbers isn't a problem.

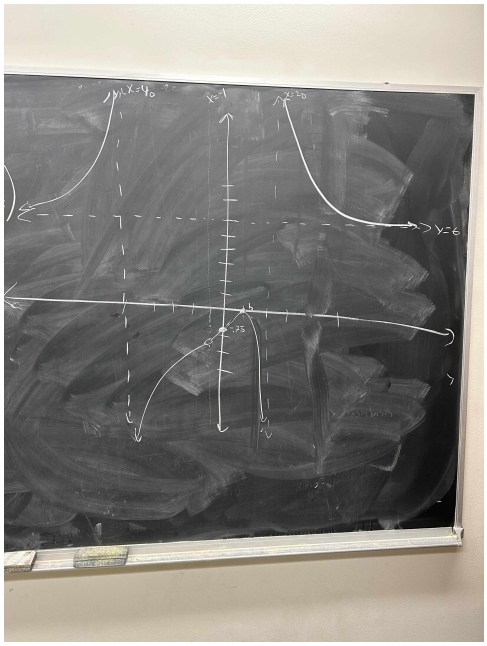


Figure 13: Use that the asymptote at $x = 2$ is odd, or start from the HA (because the HA is not zero you can do this), or plug in $b(3) = \frac{6(2)^2}{(1)(7)} = \frac{24}{7} > 0$.