MATH 31B MIDTERM REVIEW WINTER 2026

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Identifying integration techniques

Identify which integration technique to use for each

(1) $\int x^2 e^x dx$
(2) $\int \sin^3(x)\cos^4(x)dx$
(3) $\int \frac{2x^2+6}{x^3+9x-1}dx$
(4) $\int \ln(x)dx$
(5) $\int \frac{1}{x^2+6}dx$
(6) $\int \frac{x}{x^2+6}dx$
(7) $\int \frac{x^3}{(x^2+6)^{17}}$
(8) $\int \frac{1}{x^2+5x+6}dx$
(9) $\int \frac{x^2}{\sqrt{1-x^6}}dx$
(10) $\int \frac{x^3}{(1-x^2)^6}dx$

Date: February 4, 2026.

L'hopital's and various indeterminate forms

Identify the type of indeterminate form and solve the limits

(1) $\lim_{x\to 2} \frac{x^2-4x+4}{\ln(x)-\ln(2)}$
(2) $\lim_{x\to +\infty} \frac{x^2}{x+1} - \frac{x^2}{x+2}$
(3) $\lim_{x\to 0}(x)^{4x^2}$
(4) $\lim_{x\to 0}(x+1)^{x^{-2}}$

Comparing Growth at infinity 'dominating'

Is $f >> g$, $g >> f$ or neither? $f = x^2 \ln(x)$, $g(x) = x \ln(x)^3$

A P.F.D. EXAMPLE

$$\int \frac{1}{x^2+5x+6}dx = \int \frac{1}{(x+2)(x+3)}dx$$

We can then want that $\frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$ and thus we have that $1 = A(x+3) + B(x+2)$ we get that $A = -B$ and $1 = 3A + 2B$ so we have that $A = 1$ and $B = -1$, returning to the integral we have

$$\int \frac{1}{x^2+5x+6}dx = \int \frac{1}{x+2}dx - \int \frac{1}{x+3}dx = \ln(x+2) - \ln(x+3) + C$$

A trig-sub example

$$\int \frac{x^3}{(x^2+6)^{17}}dx$$

Let $x = \sqrt{3}\tan(\theta)$ so $dx = \sqrt{3}\sec^2(\theta)$ so we get $\int \frac{\sqrt{3}\tan(\theta)(3+3\sec^2\theta))}{(\sqrt{3}\sec(\theta))^{34}}\sqrt{3}\sec(\theta)^2 d\theta$ now let $u = \sec(\theta)$ and $du = \sec(\theta)\tan(\theta)d\theta$ and so we have

$$\int \frac{1}{\sqrt{3}^{30}}\frac{1}{u^{32}} + \frac{-1}{\sqrt{3}^{32}}\frac{1}{u^{33}}du$$

this is something we can integrate. The final step will be to plug back into x and for that we recall $u = \sec(\theta)$ and $x = \sqrt{3}\tan(\theta)$ so we have that $\theta = \arctan(\frac{x}{\sqrt{3}})$ and so $u = \sec(\arctan(\frac{x}{\sqrt{3}})) = \frac{\sqrt{3}}{\sqrt{x^2+3}}$.

Answer Key

1

(1) int by parts $u = x^2$ $dv = e^x$
(2) let $\sin^2 = (1 - \cos^2)$ then $u = \cos(x)$
(3) u-sub $u = x^3 + 9x - 1$
(4) int by parts $u = \ln(x)$ $dv = 1$
(5) make into an arctan integral, multiply top and bottom by 1/6
(6) u-sub $u = x^2 + 6$ or trig sub $\sqrt{6}\tan(\theta) = x$
(7) trig sub $\sqrt{6}\tan(\theta) = x$
(8) factor bottom then pfd
(9) make into a arcsin integral let $u = x^3$
(10) trig sub $\sec(\theta) = x$.

2.1

$$\begin{aligned}\lim_{x\to 2} \frac{x^2 - 4x + 4}{\ln(x) - \ln(2)} &\to \frac{0}{0} \\ &=^{LH} \lim_{x\to 2} \frac{2x-4}{\frac{1}{x}} \\ &= \frac{0}{1/2} = 0.\end{aligned}$$

2.2

$$\begin{aligned}\lim_{x\to+\infty} \frac{x^2}{x+1} - \frac{x^2}{x+2} &\to \infty - \infty \\ &= \lim_{x\to+\infty} \frac{x^2(x+2) - x^2(x+1)}{(x+1)(x+2)} \\ &= \lim_{x\to+\infty} \frac{x^3 + 2x^2 - x^3 - x^2}{x^2 + 3x + 2} \\ &= \lim_{x\to+\infty} \frac{x^2}{x^2+3x+2} = 1\end{aligned}$$

Note: This was an easier problem where you did not need L'hopitals often after simplifying you will want to apply LH to solve the resulting limits.

2.3

$$\begin{aligned}\lim_{x\to 0} (x)^{4x^2} &\to 0^0 \\ &= e^{\lim_{x\to 0} \ln((x)^{4x^2})} \\ &= e^{\lim_{x\to 0} (4x^2 \ln(x))} \\ &= e^{\lim_{x\to 0} \frac{\ln(x)}{1/(4x^2)}} \\ &=^{LH} e^{\lim_{x\to 0} \frac{1/x}{-1/(2x^3)}} \\ &= e^{\lim_{x\to 0} 2x^2} \\ &= e^0 = 1.\end{aligned}$$

2.4

$$\begin{aligned}\lim_{x\to 0}(x+1)^{x^{-2}} &\to 1^{\infty}\\ &= e^{\lim_{x\to 0} x^{-1}\ln(x+1)}\\ &= e^{\lim_{x\to 0}\frac{\ln(x+1)}{x}}\\ &= e^{\lim_{x\to 0}\frac{\frac{1}{x+1}}{1}}\\ &= e^1 = e.\end{aligned}$$

3.1 Consider $\lim_{x\to+\infty}\frac{f(x)}{g(x)} = \lim_{x\to+\infty}\frac{x^2\ln(x)}{x\ln(x)^3} = \lim_{x\to+\infty}\frac{x}{\ln(x)^2} =^{LH} \lim_{x\to+\infty}\frac{1}{2x\ln(x)\frac{1}{x}} = \lim_{x\to+\infty}\frac{1}{2\ln(x)} = 0$ so $f << g$.