

MATH 31B FINAL REVIEW WINTER 2026

JOHN HOPPER

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COMPUTING DERIVATIVES OF POWER SERIES AT CENTER

Compute the following derivatives

- (1) $f(x) = \sum_{n=2}^{+\infty} \frac{(x+3)^n (n+4)}{(n-2)!}$ find $f^{(35)}(-3)$
- (2) $f(x) = \sum_{n=38}^{+\infty} \frac{(x+3)^n (n+4)}{(n-2)!}$ find $f^{(35)}(-3)$
- (3) $f(x) = \sum_{n=0}^{+\infty} \frac{(x-1)^{2n+1} (-1)^n}{n!}$ find $f^{(101)}(1)$
- (4) $f(x) = \sum_{n=0}^{+\infty} \frac{(x-1)^{2n+1} (-1)^n}{n!}$ find $f^{(100)}(1)$
- (5) $f(x) = \sum_{n=0}^{+\infty} \frac{x^{(n^2)}}{20^n (n^2+2)!}$ find $f^{(100)}(0)$

FINDING TAYLOR SERIES

Find the Taylor Series of the following functions. Does the interval of convergence of the series match the domain of the function?

- (1) $\sin(2(x-4)^3)$ around $c = 4$
- (2) $\ln(2-x) + \frac{1-x^2}{2}$ around $c = 1$
- (3) $\int x e^{3x^2} dx$ around $c = 0$
- (4) $2x \sin(3x) + 3x^2 \cos(3x)$ around $c = 0$
- (5) $\frac{3(x-1)}{6-2x}$ around $c = 1$

TAYLOR'S THEOREM/APPROXIMATION

Compute the Taylor polynomial and estimate the error using Taylor's approximation.

- (1) Find the degree 3 Taylor polynomial of \sqrt{x} around $c = 1$ and use it to approximate $\sqrt{2}$ up to an error of 0.05. *This is likely more algebra than I would expect on an exam, but it is fully doable on pen and paper without a calculator*
- (2) Find the degree 1 Taylor polynomial of e^{4x} around $x = 0$ and approximate e^2 using the fact that $e < 3$.

INTERVALS OF CONVERGENCE

Find the intervals of convergence for each power series:

$$(1) \sum_{n=0}^{+\infty} \frac{2^{n+1}(x-\pi)^n(-1)^n}{n^2}$$

$$(2) \sum_{n=0}^{+\infty} \frac{2n!(x-4)^n}{18^n(n+2)}$$

$$(3) \sum_{n=0}^{+\infty} \frac{n!x^n}{(2n)!}$$

ANSWER KEY

1

- (1) (35) · (34) · (39)
- (2) 0
- (3) $\frac{(101!) \cdot (-1)^{50}}{100!} = 101$
- (4) 0
- (5) $\frac{(100!)}{20^{10}(102!)} = \frac{1}{20^{10} \cdot 102 \cdot 101}$.

2

- (1) $\sum_{n=0}^{+\infty} \frac{(x-4)^{6n+3} 2^{6n+3} (-1)^n}{(2n+1)!}$ Yes the domain and the interval of convergence are both \mathbb{R} .
- (2) $\sum_{n=3}^{+\infty} \frac{(x-1)^n}{n}$ No the domain of $\ln(2-x) + \frac{1-x^2}{2}$ is $x < 2$ but the interval of convergence is only $[0, 2)$.
- (3) $\sum_{n=0}^{+\infty} \frac{3^n x^{2n+2}}{(2n+2)n!}$, yes the domain and the interval of convergence are both \mathbb{R} .
- (4) Note that $2x \sin(3x) + 3x^2 \cos(3x) = \frac{d}{dx}(x^2 \sin(3x))$ and so $\sum_{n=0}^{+\infty} \frac{x^{2n+2} 3^{2n+1} (-1)^n (2n+3)}{(2n+1)!}$, yes the domain and the interval of convergence are both \mathbb{R} .
- (5) $\sum_{n=0}^{\infty} \frac{3(x-1)^{n+1}}{2^{n+2}}$, no the domain of $\frac{3(x-1)}{6-2x}$ is $x \neq 3$ but the interval of convergence is $(-1, 3)$

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- (1) Let $f(x) = \sqrt{x}$, note $f'(x) = \frac{1}{2}x^{-1/2}$, $f''(x) = -\frac{1}{4}x^{-3/2}$, $f'''(x) = \frac{3}{8}x^{-5/2}$, $f''''(x) = -\frac{15}{16}x^{-7/2}$, so the Taylor polynomial of degree 3 polynomial of f at 1 is $P(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$. So we can see that the approximation for $\sqrt{2}$ is $1 + \frac{1}{2} - \frac{1}{8} + \frac{1}{16} = \frac{23}{16} = 0.4375$. We can now estimate the error, note $|f''''(x)| = |-\frac{15}{16}x^{-7/2}| \leq \frac{15}{16}$ when $x \geq 1$. So we can see that the error of our computation is $(2-1)^4 \frac{15}{16 \cdot 4!} = \frac{5}{128} < 0.05$ so our estimate 0.4375 is within the desired error.
- (2) Let $f(x) = e^{4x}$, note $f'(x) = 4e^{4x}$ and $f''(x) = 16e^{4x}$. We can find that $f(0) = 1$ and $f'(0) = 4$ so are degree 1 Taylor polynomial is $P(x) = 1 + 4x$. We can then see that the error is bounded by $|16e^{4x}|$ where $x \in [0, 1/2]$ and so plugging 3 for e we get $|16e^{4x}| \leq 16(3^2) = 16 \cdot 9$ for $x \in [0, 1/2]$. We can then see that the error of the estimate is $16 \cdot 9(1/2)^2 \frac{1}{2!} = 2 \cdot 9 = 18$. We we have that $e^2 = 3 \pm 18$.

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- (1) $[\pi - 0.5, \pi + 0.5]$
- (2) $\{4\}$ only the one point
- (3) $(-\infty, \infty)$