Some 32A Practice Problems

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Particle Motion

1. *A particle moves along a path given by the curve:*

$$\mathbf{r}(t) = \left\langle 2t^2 + 1,\, 2e^t + 2,\, t^3 \right\rangle$$

What are the normal and tangential components of acceleration when the particle is moving in the direction of the y-axis?

Projectile Motion

1. *One night, Joe sneaks onto the roof of Swift Hall — which is* 10 *meters tall — and throws his shoe at an angle of* 30 *degrees to the horizontal with an initial speed of* $2\,m/s$*. How fast is his shoe going when it hits the ground? Take gravity to be* $10\,m/s^2$*.*

2. *Joe is having a little too much fun on Dillo Day, makes a poor judgement call, and shoots a firework off in front of Lunt Hall. The firework is launched in the western direction (in the direction of the negative* x *axis) at an angle of* 60 *degrees, with an initial speed of* $20\,m/s$*. The wind gives the firework a northerly acceleration of* $3\,m/s^2$*. Take gravity to be* $10\,m/s^2$*. How far away from Lunt does the firework land?*

3. *(Challenging Problem) Joe is about to take the halfcourt shot at the NU-Wisconsin basketball game. The basketball hoop is* 3 *meters high, and is* 28 *meters from half court. Unfortunately, Joe hasn't played basketball since* 8*th grade and has no idea how hard to shoot the ball, so he shoots at an angle of* 45 *degrees with an initial speed of* $20\,m/s$*. Also, due to the faltering structural integrity of Welsh-Ryan Arena, there is a draft that gives the ball a left-ward acceleration of* $1\,m/s$*. Take gravity to be* $10\,m/s^2$*, and suppose for the sake of simplicity that Joe shoots from the point* $(0,0,0)$*. How close does the ball come to going in?*

Multivariable Functions

1. *Let* $f(x,y) = x + y^2$*. Determine the domain of* f *and draw a contour plot with three level curves corresponding to the values* -1*,* 0*, and* 1*.*

2. *Let* $f(x,y) = \frac{x^2-y^2}{x+y}$*. Describe the domain of the function, and draw a contour plot.*

3. *Describe the level surfaces of the function* $f(x,y,z) = x^2 + y^2 + z^2$*.*

Multivariable Limits and Continuity

1. *(Technique: Continuity) Evaluate the following limit:*

$$\lim_{(x,y)\to(0,0)} \frac{x^{2-y}+3}{\cos(xy)+1}$$

2. *(Technique: Different Path Approaches) Evaluate the following limit:*

$$\lim_{(x,y)\to(0,0)} \frac{2x^3-xy-3y^3}{3x^3-2y^3}$$

3. *(Technique: Different Path Approaches) Evaluate the following limit:*

$$\lim_{(x,y)\to(0,0)} \frac{x^2+xy+y^2}{xy+x^2+2y}$$

4. *(Technique: Different Path Approaches) Evaluate the following limit:*

$$\lim_{(x,y)\to(0,0)} \frac{xy}{2x^2+3y^2}$$

5. *(Technique: Different Path Approaches) Evaluate the following limit:*

$$\lim_{(x,y)\to(0,0)} \frac{x^4y}{x^8+y^2}$$

6. *(Technique: Factoring) Evaluate the following limit:*

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^2+y}$$

7. *(Technique: Conjugate) Evaluate the following limit:*

$$\lim_{(x,y)\to(0,0)} \frac{x^3+xy+y^2}{2-\sqrt{x^3+xy+y^2+4}}$$

8. *(Technique: Squeeze Theorem / Comparison) Evaluate the following limit:*

$$\lim_{(x,y)\to(0,0)} \frac{x^4y^2}{x^4+3y^6}$$

9. *(Technique: Polar Coordinates) Evaluate the following limit:*

$$\lim_{(x,y)\to(0,0)} \frac{2xye^{\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}$$

10. *(Technique: Polar Coordinates) Evaluate the following limit:*

$$\lim_{(x,y)\to(0,0)} \frac{2xye^{\sqrt{x^2+y^2}}}{x^2+y^2}$$

11. *Is the following function continuous at the point* $(0,0,0)$*?*

$$f(x,y,z) = \begin{cases} \frac{xy+yz+z^2}{x+y+z^2} & (x,y,z)\neq(0,0,0) \\ 1 & (x,y,z)=(0,0,0) \end{cases}$$

Partial Derivatives

1. *Let* $f(x,y) = \frac{e^{xy}}{x^2+y^2}$. *Compute* $f_x(x,y)$ *and* $f_y(x,y)$.

2. *Let* $g(r,\theta,z) = r^2 \cos 2z\theta + zr\theta - 2^{r-z}$. *Compute* $g_z(1, \frac{\pi}{4}, 2)$.

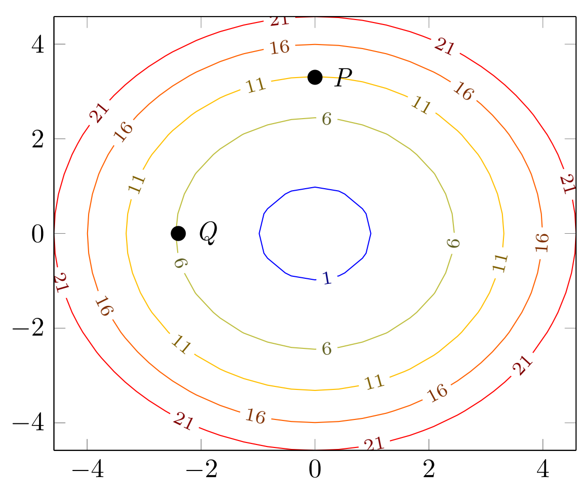
3. *Joe just bought an Old Fashioned Buttermilk donut and a Valrhona Triple Chocolate donut from Firecakes, and is trying to decide which one to eat first. After extensive testing and data collection, Joe has modeled his happiness as a function of consumption of donuts. Let* H *denote happiness,* B *denote the number of Old Fashioned Buttermilk donuts consumed, and* C *denote the number of Valrhona Triple Chocolate donuts consumed. Then*

$$H(B,C) = e^{3B} + (B+1)(C+1)^3 + e^{\frac{C}{2}}$$

If Joe only has time to take a single bite of one donut, which one should he eat?

4. *Let* $f(x,y) = \ln(x^2+y^2+1)$. *Prove that* $f_{xy} = f_{yx}$.

5. *Consider the contour plot of the function* $f(x,y)$ *below.*



Determine whether each of the following quantities is positive, negative, or zero:

(a) $f_x(P)$

(b) $f_y(P)$

(c) $f_{yy}(P)$

(d) $f_y(Q)$

(e) $f_x(Q)$

(f) $f_{xx}(Q)$

Tangent Planes and Approximation

1. *Let* $f(x,y) = 2xy + 2^{xy}$. *Find the equation of the tangent plane to* f *at the point* $(1,2)$.

2. *Let $f(x,y,z) = xy + xz + yz$. Find the linear approximation to f at the point $(1,2,3)$.*

3. *Let $f(x,y) = x^3y^3 + x + y$. Find the linear approximation and quadratic approximation to f at the point $(1,1)$. Find Δz in moving from the point $(1,1)$ to $(2,2)$. Estimate Δz with the linear and quadratic approximation.*

The Chain Rule

1. *Let $f(x,y) = x^2y + xy^2$ and $x(u,v) = 2uv$ and $y(u,v) = u^2 + v^2$. Consider f as a function of u and v by $f(x(u,v), y(u,v))$. Use the chain rule to compute $\frac{\partial f}{\partial u}$.*

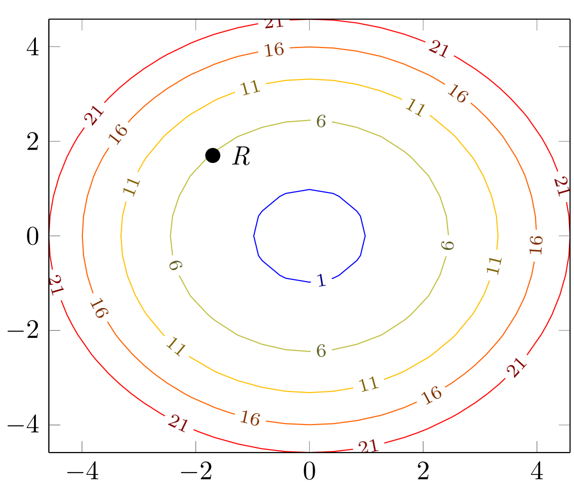
2. *Let $g(t) = f(x(t), y(t))$, where $x(t) = t^2$ and $y(t) = 2t + 1$. Use the following table of values to compute $g'(1)$.*

$f_x(1,1) = 2$	$f_y(1,1) = 3$
$f_x(2,1) = 4$	$f_y(2,1) = -2$
$f_x(1,3) = -1$	$f_y(1,3) = 5$

Directional Derivatives and Gradients

1. *Let $f(x,y,z) = x^2y - 2xz$. Compute the gradient of f, and find the directional derivative of f in the direction of the line $y = x = z$.*

2. *Consider the contour plot of the function $f(x,y)$ below.*



Draw the gradient vector at point R, and determine whether the directional derivative in the southeast direction is positive, negative, or 0.

3. *Consider the surface* $y = x^2 + z^2 - 2$. *Determine at what points on the surface, if any, the tangent plane is parallel to the* $x - z$ *plane.*

4. *After getting home from the Northwestern - Purdue game, Joe had a choice - he could either stay home all day and watch football, or go to the library and do work, or do some combination of both. Joe modeled his happiness* H *as a function of time spent in the library* (L) *and time spent watching football* (F)*:*

$$H = F - L$$

What fraction of his time should he spend doing each to be as happy as possible?