A Graphical Representation of Integration in 32B

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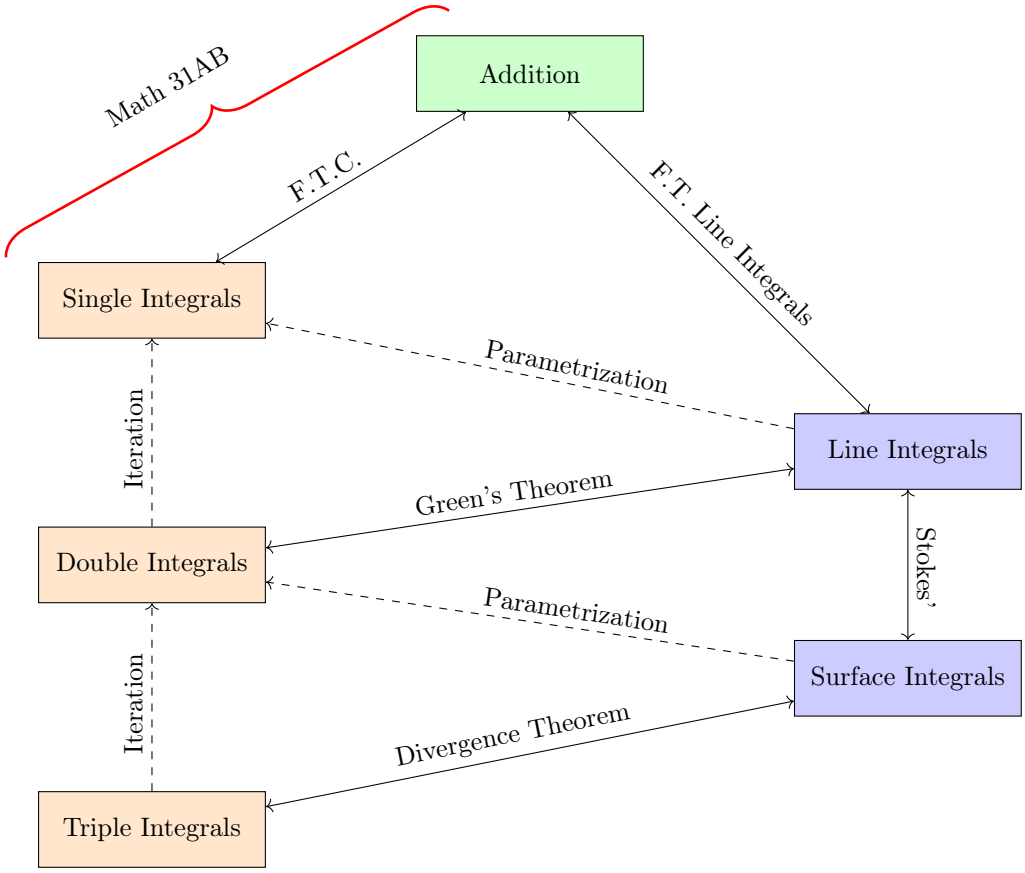
Mutlivariable calculus is a zoo of integrals, from double and triple integrals to line and surface integrals. Packaged with these integrals is a host of fundamental theorems such as Green's theorem and Stokes' theorem. Understanding them all can be confusing The following graphic is an (over-simplified) visual representation of the big picture.



Explaining the Graphic

Each box represents an integral. The arrows between the boxes represents a way to transition from one to the other. For example, one way to evaluate a line integral is to use a parametrization to convert it into a single variable integral. Or one can convert a surface integral to a triple integral using the divergence theorem. The boxes and arrows above summarize these relationships. Obviously this is a simplification, because there are conditions that need to be satisfied to apply theorems like Green's theorem and the like.

At the end of the day, the only way we can *really* evaluate any kind of integral is turn it into an addition problem. For example, to evaluate the single integral

$$\int_0^1 x^2\,dx$$

we use the fundamental theorem of calculus to turn it into an addition problem:

$$\int_0^1 x^2\,dx = \frac{x^3}{3}\bigg|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}.$$

Thus, in order to evaluate *any*kind of integral is to follow the above arrows in some manner to eventually turn it into an addition problem!