Math 234 Practice Problems

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1. *Evaluate the line integral* $\int_C xy\,dx + y^2\,dy + yz\,dz$, *where* C *is the line segment from the point* $(1, 0, -1)$ *to* $(3, 4, 2)$.

2. *Evaluate the following integral by making a change of variables:*
$$\iint_R \frac{x-2y}{3x-y}\,dA$$
where R *is the parallelogram bounded by* $x - 2y = 0$, $x - 2y = 4$, $3x - y = 1$, *and* $3x - y = 8$.

3. *Use Green's Theorem to evaluate the the line integral*
$$\int_C \sqrt{1+x^3}\,dx + 2xy\,dy$$
where C *is boundary of the triangle with vertices* $(0,0)$, $(1,0)$, *and* $(1,3)$, *oriented counterclockwise.*

4. *Find the surface area of the part of the paraboloid* $x = y^2 + z^2$ *that lies inside the cylinder* $y^2 + z^2 = 9$.

5. *Let* C *be a simple, closed curve that lies in the plane* $x + y + z = 1$. *Show that the line integral*
$$\int_C z\,dx - 2x\,dy + 3y\,dz$$
only depends on the area of the region enclosed by C *and not on the shape of* C *or its position in the plane.*

6. *Let* $\mathbf{F}(x,y,z) = \langle xy,\, 3y,\, 5y\rangle$, *and let* C *be the curve of intersection of the plane* $x + z = 5$ *and the cylinder* $x^2 + y^2 = 81$. *Compute* $\oint_C \mathbf{F}\cdot d\mathbf{r}$.

7. *Let* $\mathbf{F}(x,y,z) = \langle \sin y,\, x\cos y + \cos z,\, -y\sin z\rangle$. *Let* C *be the curve given by* $\mathbf{r}(t) = \langle \sin t,\, t,\, 2t\rangle$ *for* $0 \le t \le 2\pi$.

   (a) *Is* $\mathbf{F}$ *conservative?*

   (b) *Evaluate* $\int_C \mathbf{F}\cdot d\mathbf{r}$.

8. *Use Stokes' Theorem to evaluate* $\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S}$, *where* $\mathbf{F}(x,y,z) = \left\langle \arctan(x^2yz^2),\, x^2y,\, x^2z^2\right\rangle$ *and* S *is the cone* $x = \sqrt{y^2+z^2}$ *from* $0 \le x \le 2$ *oriented in the direction of the positive* x*-axis.*

9. *Fix* $a > 0$. *Let* $\mathbf{F} = \langle xz,\, x,\, y\rangle$, *and let* S *be the surface given by:*
$$x^2 + y^2 + z^2 = a^2 \qquad y \ge 0$$
Compute: $\iint_S \mathbf{F}\cdot d\mathbf{r}$

10. *Use the Divergence Theorem to evaluate* $\iint_S (2x + 2y + z^2)\,dS$ *where* S *is the sphere* $x^2 + y^2 + z^2 = 1$.