Suppose I’m faced with an infinite series $\sum a_n$ and some higher power wants me to determine whether or not the series converges or diverges. If I don’t know what to do, here are the questions I ask myself:

1. **Do the terms look like they don’t go to zero?**
   
   If so, try the *divergence test*.

2. **Is the series of a special form?**
   
   By that I mean:
   
   (a) **Does it have only terms with numbers raised to \( n \)th powers?**
   
   If so, it might be *geometric*.

   (b) **Is it a \( p \)-series?**
   
   If so...it’s a *\( p \)-series*.

   (c) **Do I see a \((-1)^n\) or some other alternating behavior?**
   
   If so, try the *alternating series test*. If the alternating series test fails, it probably failed because the terms don’t go to zero, so try the *divergence test* again.

   (d) **Is it a rational function, or am I asked to evaluate it?**
   
   If so, it might be *telescoping*. Do partial fractions if necessary and compute partial sums.

3. **Do I see a factorial, or a combination of exponentials and polynomials?**
   
   If so, try the *ratio test*.

4. **Do I see a complicated exponential expression?**
   
   If so, try the root test.

5. **Are there obvious dominant terms that I can isolate, or is there fluff I can throw away?**
   
   If so, try one of the comparison tests. Generate a term to compare to by isolating the dominant behavior.

6. **Are there bounded trig functions or logs?**
   
   If so, maybe try direct comparison by bounding the trig functions or using a comparison like $\ln n \leq n^a$.

7. **Does the sequence look like a function I can integrate, or are there logs?**
   
   If so, try the integral test. The integral test together with a substitution can be helpful for many series involving logs, if nothing else works.

8. **All of that failed, now what.**
   
   A last ditch resort is to go back and try the limit comparison test again, but with a clever or non-obvious choice. If you know anything about Taylor polynomials, this can sometimes be enlightening.