CONCEPTUAL TRUE OR FALSE 32A QUESTIONS

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True/false questions are one of the best ways to test your understanding of a concept. (They're also fun!) Even though the focus of 32A is *computation* and not necessarily the theory, being able to think and reason abstractly indirectly translates to better computational ability.

The first section contains the statements, the second section contains *only the true or false answers* for if you get stuck and want the answer to help lead you to an explanation, and the third section contains explanations. I will (hopefully) add more and update solutions as the quarter goes on.

Contents

Problems

For each of the following statements, indicate TRUE or FALSE. If the answer is TRUE, give some reasoning. If the answer is FALSE, provide a counter example.

1. Suppose that the curvature function $\kappa(t)$ of a curve is constant. Then the curve is a circle.

2. Suppose that $\mathbf{r}(\mathbf{t})$ and $\mathbf{r}''(t)$ are parallel. Then $\mathbf{r}(t) \times \mathbf{r}'(t)$ is constant.

3. The area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$ is the same as the area of the parallelogram spanned by $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$.

4. Suppose that $\mathbf{r}(t)$ has constant speed 2. Then an arc length parameterization for $\mathbf{r}(t)$ is $\frac{1}{2}\mathbf{r}(t)$.

5. There exists a curve $\mathbf{r}(t)$ with constant speed 2 such that $\frac{1}{2}\mathbf{r}(t)$ is an arc length parameterization of the same curve.

6. The underlying space curve of $\mathbf{r}(t)$ and $\mathbf{r}(t^2)$ are the same.

7. Suppose that two parametric lines $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ have the same direction vector. Then there is some t and s such that $\mathbf{r}_1(t) \times \mathbf{r}_2(s) = \mathbf{0}$.

8. Suppose that $\mathbf{r}(s)$ is an arc length parameterization. Then $\mathbf{r}'(s)$ and $\mathbf{r}''(s)$ are orthogonal.

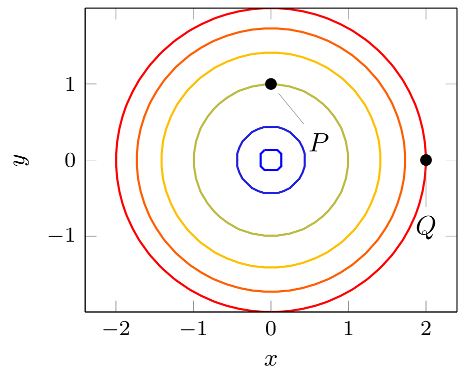
9. The volume of the parallelopiped spanned by $\mathbf{a}, \mathbf{b}, \mathbf{c}$ is $\|\mathbf{a}\| \, \|\mathbf{b}\| \, \left\|\text{proj}_{\mathbf{a}\times\mathbf{b}}\mathbf{c}\right\|$.

10. Suppose that $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ are two parameterizations of the same curve such that $\mathbf{r}_1(0) = \mathbf{r}_2(0)$ and $\mathbf{r}_1(1) = \mathbf{r}_2(1)$. Then
$$\int_0^1 \|\mathbf{r}_1'(t)\| \; dt = \int_0^1 \|\mathbf{r}_2'(t)\| \; dt.$$

11. Fix a nonzero vector $\mathbf{n}$. Suppose that P and Q are different points. Then the plane through P with normal vector $\mathbf{n}$ is different than the plane through Q with normal vector $\mathbf{n}$.

12. Suppose that $x = x(t)$ and $y = y(t)$ are parametric equations such that $y'(0) = 0$. Then the tangent line to the curve at the point $(x(0), y(0))$ is horizontal.

13. Suppose that $f(x, y)$ with the following contour plot:



    (a) The vector $\nabla f(P)$ points directly north.
    (b) The vector $\nabla f(Q)$ has greater magnitude than $\nabla f(P)$.
    (c) The values $f(P)$ and $f(Q)$ are different, i.e., $f(P) \neq f(Q)$.
    (d) The function f has a critical point at $(0, 0)$.
    (e) The partial derivative $f_x(P)$ is 0.
    (f) The partial derivative $f_y(P)$ is 0.

14. Suppose that the curvature of a curve $\mathbf{r}(t)$ at time 0 is 0, that is, $\kappa(0) = 0$. Then $\mathbf{r}''(0) \cdot \mathbf{T}(0) = \|\mathbf{r}''(0)\|$.

15. Suppose that $\mathbf{r}(t)$ has a nonzero acceleration vector at time 0, that is, $\mathbf{r}''(0) \neq \mathbf{0}$. Then $\kappa(0) \neq 0$.

Answers

1. Suppose that the curvature function $\kappa(t)$ of a curve is constant. Then the curve is a circle.

   Answer. False. □

2. Suppose that $\mathbf{r}(\mathbf{t})$ and $\mathbf{r}''(t)$ are parallel. Then $\mathbf{r}(t) \times \mathbf{r}'(t)$ is constant.

   Answer. True. □

3. The area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$ is the same as the area of the parallelogram spanned by $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$.

   Answer. False. □

4. Suppose that $\mathbf{r}(t)$ has constant speed 2. Then an arc length parameterization for $\mathbf{r}(t)$ is $\frac{1}{2}\mathbf{r}(t)$.

   Answer. False. □

5. There exists a curve $\mathbf{r}(t)$ with constant speed 2 such that $\frac{1}{2}\mathbf{r}(t)$ is an arc length parameterization of the same curve.

   Answer. True. □

6. The underlying space curve of $\mathbf{r}(t)$ and $\mathbf{r}(t^2)$ are the same.

   Answer. False. □

7. Suppose that two parametric lines $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ have the same direction vector. Then there is some t and s such that $\mathbf{r}_1(t) \times \mathbf{r}_2(s) = \mathbf{0}$.

   Answer. True. □

8. Suppose that $\mathbf{r}(s)$ is an arc length parameterization. Then $\mathbf{r}'(s)$ and $\mathbf{r}''(s)$ are orthogonal.

   Answer. True. □

9. The volume of the parallelopiped spanned by $\mathbf{a}, \mathbf{b}, \mathbf{c}$ is $\|\mathbf{a}\| \, \|\mathbf{b}\| \, \left\|\text{proj}_{\mathbf{a}\times\mathbf{b}}\mathbf{c}\right\|$.

   Answer. False. □

10. Suppose that $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ are two parameterizations of the same curve such that $\mathbf{r}_1(0) = \mathbf{r}_2(0)$ and $\mathbf{r}_1(1) = \mathbf{r}_2(1)$. Then

$$\int_0^1 \|\mathbf{r}_1'(t)\| \; dt = \int_0^1 \|\mathbf{r}_2'(t)\| \; dt.$$

   Answer. False. □

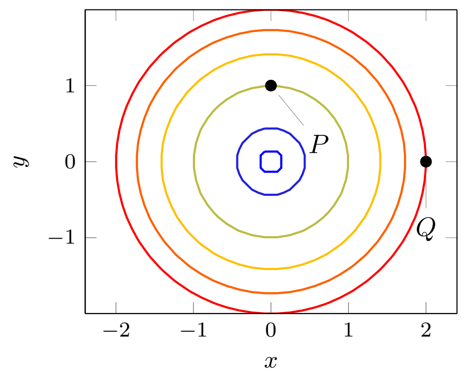
11. Fix a nonzero vector $\mathbf{n}$. Suppose that P and Q are different points. Then the plane through P with normal vector $\mathbf{n}$ is different than the plane through Q with normal vector $\mathbf{n}$.

Answer. False. □

12. Suppose that $x = x(t)$ and $y = y(t)$ are parametric equations such that $y'(0) = 0$. Then the tangent line to the curve at the point $(x(0), y(0))$ is horizontal.

Answer. False. □

13. Suppose that $f(x, y)$ with the following contour plot:



(a) The vector $\nabla f(P)$ points directly north.
(b) The vector $\nabla f(Q)$ has greater magnitude than $\nabla f(P)$.
(c) The values $f(P)$ and $f(Q)$ are different, i.e., $f(P) \neq f(Q)$.
(d) The function f has a critical point at $(0, 0)$.
(e) The partial derivative $f_x(P)$ is 0.
(f) The partial derivative $f_y(P)$ is 0.

Answer.

(a) False.
(b) False.
(c) False.
(d) True.
(e) True.
(f) False.

□

14. Suppose that the curvature of a curve $\mathbf{r}(t)$ at time 0 is 0, that is, $\kappa(0) = 0$. Then $\mathbf{r}''(0) \cdot \mathbf{T}(0) = \|\mathbf{r}''(0)\|$.

Answer. True. □

15. Suppose that $\mathbf{r}(t)$ has a nonzero acceleration vector at time 0, that is, $\mathbf{r}''(0) \neq \mathbf{0}$. Then $\kappa(0) \neq 0$.

Answer. False. □

Answers with explanations

(will add eventually)