32B Week 1: Pep talk / double integrals via R.S.'s

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My website has important info (OH, videos)
Math is about communicating

Example 1.5. Let \( f(x, y) = xy^2 \) and \( E = [1, 2] \times [2, 4] \). Estimate the double integral

\[
\iint_E f(x, y) \, dA
\]

using a Riemann sum with 4 rectangles.

**Solution.**

\[
\iint_E f(x, y) \, dA = \lim_{n \to \infty} \sum_{i=1}^{2} \sum_{j=1}^{2} f(x_{ij}, y_{ij}) \Delta x_j \Delta y_i
\]

Consider the x partition
\[1 \leq 1.5 \leq 2\]
Consider the y partition
\[2 \leq 3.5 \leq 4\]

Vol of box 1

So

\[
\iint_E f(x, y) \, dA \approx f(3,4) (2-1.5)(4-3.5) + f(3,5) (2-1.5)(3.5-2)
\]
\[
+ f(1,2) (1.5-1)(3.5-2) + f(1,4) (1.5-1)(4-3.5)
\]
\[
= (2)(4)(0.5)(0.5) + (2)(3)(0.5)(0.5) + (1)(2)(0.5)(1.5) + (1)(4)(0.5)(0.5)
\]
\[
= 29.5
\]
Example 1.6. Let $E$ be any region in the $(x, y)$ plane. Interpret the meaning of

$$\int\int_E 1 \, dA.$$ 

**Solution.**

$$\int\int_E 1 \, dA = \text{area of } E.$$ 

**Explanation 1:**

$$\int\int_E 1 \, dA = \text{volume of region under 1 over } E$$

$$= (\text{area of base})(\text{height})$$

$$= \text{area of } E.$$ 

**Explanation 2:**

[Diagram with grid and notation $\sum \Delta x \Delta y$.]