Math 32A Practice Midterm 2

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Name: _____
ID: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. For each of the following statements, answer TRUE or FALSE. No justification required! Read carefully!!

   (a) (2 points) If $\int_0^1 \mathbf{r}'(t)\,dt = \mathbf{0}$, then $\int_0^1 \|\mathbf{r}'(t)\|\,dt = 0$.

   (b) (2 points) If $\|\mathbf{r}'(t)\| = 2$, then $\kappa(t) = \frac{1}{4}\|\mathbf{r}''(t)\|$.

   (c) (2 points) Suppose that $\mathbf{r}(t)$ is a regular parameterization (this just means that $\mathbf{r}'(t) \neq \mathbf{0}$ for all t) for $-\infty < t < \infty$ such that $\kappa(t) = 0$, then

   $$\lim_{t\to\infty} \|\mathbf{r}(t)\| = \infty.$$

   (d) (2 points) The direction of steepest descent of a function f is given by $-\nabla f$.

   (e) (2 points) If $\mathbf{r}(s)$ is an arc length parameterization, then $D_{\mathbf{r}'(s)} f(\mathbf{r}(s)) = \nabla f(\mathbf{r}(s)) \cdot \mathbf{r}'(s)$.

   Answers:

   (a)

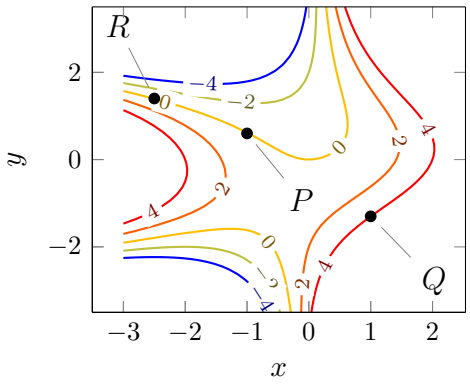
   (b)

   (c)

   (d)

   (e)

2. Consider the following contour plot of a function $f(x, y)$.



(a) (2 points) Sketch the gradient vector $\nabla f(P)$ on the plot above. (Don't worry about drawing the correct length, just get the correct direction.)

(b) (2 points) Let $\mathbf{v} = \langle 0, 1 \rangle$. Is the directional derivative $D_{\mathbf{v}} f(Q)$ positive, negative, or zero?

(c) (2 points) Let $\mathbf{r}(t)$ be a parameterization for the level curve passing through the point Q. Suppose that $\mathbf{r}(0) = Q$. Let $\mathbf{u} = \frac{\mathbf{r}'(0)}{\|\mathbf{r}'(0)\|}$. Is the directional derivative $D_{\mathbf{u}} f(Q)$ positive, negative, or zero?

(d) (2 points) Which number is larger: $\|\nabla f(P)\|$ or $\|\nabla f(R)\|$?

(e) (2 points) Which number is likely closer to 0: $f_x(P)$ or $f_y(P)$?

3. Consider the function

$$f(x,y) = \frac{(x-1)x^3y}{(y-1)(x^2+y^2)}.$$

(a) (2 points) What is the domain of f?

(b) (4 points) Evaluate the limit

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

or show that it does not exist.

(c) (4 points) Evaluate the limit

$$\lim_{(x,y)\to(1,1)} f(x,y)$$

or show that it does not exist.

4. Consider the curve $y = \ln x$

   (a) (8 points) Find the point of maximum curvature on the curve.

   (b) (2 points) Write down a function $g(x, y)$ such that the curve $y = \ln x$ is a level curve for g.

5. Let $h(x, y) = x^2 + 2y^2$.

   (a) (4 points) Find the point (a, b) where the tangent plane to the graph of h is parallel to the plane $3x - 5y + 2z = 0$.

   (b) (3 points) Find the maximum possible rate of change of h at the point $(2, -1)$.

   (c) (3 points) Find a unit direction vector $\mathbf{u}$ such that $D_{\mathbf{u}}h(2, -1) = 0$.