1. Prove that \( D_x(y^T A x) = y^T A \).

2. Prove that \( D_x(x^T A x) = x^T (A + A^T) \), \( A \) square.

3. Find the gradient & Hessian of \( f(x) = x^T A x \)

   \[ \nabla f = (x^T (A + A^T))^T = (A + A^T)x, \]

   \[ D^2 f = \nabla^2 f = A + A^T. \]

4. Write multivariate Taylor expansion in terms of gradient & Hessian. \( \frac{\partial f}{\partial x} : D \to \mathbb{R}, D \subset \mathbb{R}^d \)

   \[ f(\bar{x}) = f(\bar{a}) + (\nabla f(\bar{a}))^T (x - \bar{a}) + (x - \bar{a})^T D^2 f(\bar{a})(x - \bar{a}) + O(\|x - \bar{a}\|^3). \]

5. Show that \( \sin(h) = O(h) \).

6. Show that \( 1 - \cos(h^2) = O(h^4) \).

7. Prove the matrix MVT: if \( \tilde{f} : \mathbb{R}^n \to \mathbb{R}^m \) is differentiable on \( \mathbb{R}^n \), then \( \forall \bar{x}, \bar{y} \in \mathbb{R}^n \), \( \exists M \) s.t. \( \tilde{f}(x) - \tilde{f}(\bar{y}) = M(x - \bar{y}) \).
8. Show that \( \|A\| < 1 \) implies \( \lim_{k \to \infty} A^k = 0 \).

9. Show that \( \|A\| \geq \rho(A) \).