32A1 H.W. 5 Solutions

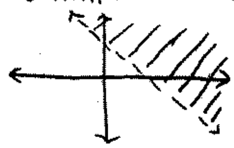
15.1

6. $f(x,y) = \ln(x+y-1)$

   (a) $f(1,1) = \ln(1+1-1) = 0$.

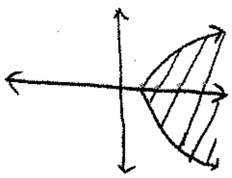
   (b) $f(e,1) = \ln(e+1-1) = 1$.

   (c) Domain: We need $x+y-1 > 0 \Rightarrow y > -x+1$



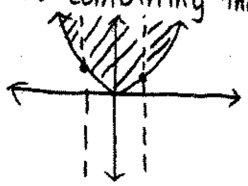
   (d) Range is the same as the range of $\ln \Rightarrow (-\infty, \infty)$.

8. Domain: $1+x-y^2 \geq 0 \Rightarrow y^2 \leq 1+x$



Range: Same as range of $\sqrt{\ } \Rightarrow [0,\infty)$.

17. Domain: We must have $y-x^2 \geq 0$ so that the $\sqrt{\ }$ is defined.
   Also $1-x^2 \neq 0$ (denominator). $y-x^2 \geq 0 \Rightarrow y \geq x^2$.
   And $1-x^2 \neq 0 \Rightarrow x \neq 1,-1$.
   So combining the conditions, $y \geq x^2$, $x \neq -1, 1$.



Range: Let $x=0$. Then varying y, we can obtain any nonnegative real number: $\frac{\sqrt{y-0^2}}{1-0^2} = \sqrt{y}$.

Now let $x=2$: $\frac{\sqrt{y-2^2}}{1-2^2} = -\frac{1}{3}\sqrt{y-4}$.

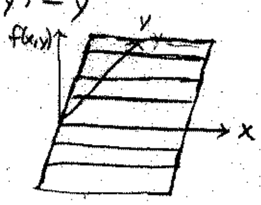
Letting y vary for $y \geq 4$, we can obtain any nonpositive real number. Then the range is $(-\infty, \infty)$.

15.1

20. We must have $16-4x^2-4y^2-z^2>0$

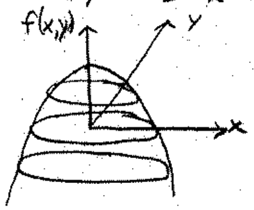
$\Rightarrow \frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{16}<1$, the set of points inside an ellipsoid.

22. $f(x,y)=y$



A plane where $y=z$.
You can think of it as taking the line $y=z$ in 2D and transporting it through all values of x.

26. $f(x,y)=3-x^2-y^2$



x^2+y^2 should be familiar to you by now as having something to do with a circle. Let $x^2+y^2=r^2$ and consider $f(r)=3-r^2$. We get a parabola. We rotate this in x,y around the z-axis ($f(x,y)$-axis) to get a paraboloid.

30. (a) VI Consider lines of constant x - say $x=0$. We should have $z=|y|$, so straight lines of slope ± 1. Only VI fits this.

(b) V If $x=0$ or $y=0$, $f(x,y)=0$. Only V fits this.

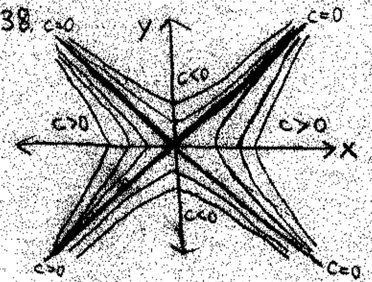
(c) I We have a maximum at $(x,y)=(0,0)$ and then $f(x,y)\to 0$ as $x,y\to\infty$. Also note that we should have perfect symmetry about $z=0$ because of the x^2+y^2 term with no other x's and y's - the telltale sign of circular cross sections along $z=$ const.

(d) IV Consider $y=0$. Then $f(x,0)=x^4$. Then note that we have symmetry: $f(x,y)=f(-x,y)$ and $f(x,y)=f(x,-y)$.

(e) II Similar to (e), but now the symmetry between $x,-x$ is gone.

15.1

30. (f) III For $y=0$, we get $f(x,0) = \sin|x|$, which is periodic.
Only III has periodicity.

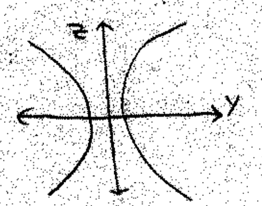
38. 

$f(x,y) = x^2 - y^2 = c$

13.6

6. $yz = 4$ gives a hyperbola in the yz-plane. This can vary over all x.

one piece:



9. (a) For $x = c$ (constant), $y^2 - z^2 = 1 - c^2$, so the traces are hyperbolas.
Similarly for $y = c$.
For $z = c$, $x^2 + y^2 = 1 + c^2$, circles.
So this is taking a hyperbola and rotating it around the z-axis.

(b) The graph should be the same with y, z flipped, so we still have a hyperboloid of one sheet.

(c) We have $x^2 + y^2 + 2y - z^2 = x^2 + (y+1)^2 - z^2 - 1 = 0$
$\Rightarrow x^2 + (y+1)^2 - z^2 = 1$.
So we have the same as in part a, but shifted -1 units in the y direction.

13.6

21. This is the basic equation of an ellipsoid, slightly disguised:

$$x^2 + \frac{y^2}{(1/2)^2} + \frac{z^2}{(1/3)^2} = 1.$$

The longest axis is in x.

VII.

22. Ditto. We get $\frac{x^2}{(1/3)^2} + \frac{y^2}{(1/2)^2} + z^2 = 1.$

IV.

23. From #9, this is a hyperboloid of one sheet. (Hyperbola rotated around y-axis)

II

24. This is the basic form of a hyperboloid of two sheets.

III.

25. Paraboloid opening up toward y^+ axis.

VI.

26. Cones opening up toward y^+,

I

27. This is an ellipse in the xz-plane. We can vary y to get a cylinder-like figure.

VIII.

28. For $z=0$, we get $y = x^2$, so the projection in the xy-plane is a parabola. We have a hyperbolic paraboloid.

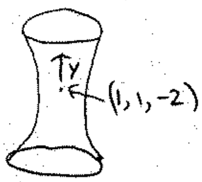
V.

36. $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0.$

Complete the square: $(x-1)^2 - 1 - (y-1)^2 + 1 + (z+2)^2 - 4 + 2 = 0$

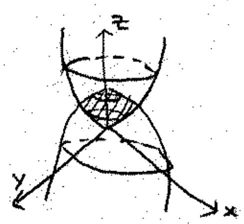
$$\Rightarrow (x-1)^2 - (y-1)^2 + (z+2)^2 = 2 \Rightarrow \frac{(x-1)^2}{(\sqrt{2})^2} - \frac{(y-1)^2}{(\sqrt{2})^2} + \frac{(z+2)^2}{(\sqrt{2})^2} = 1,$$

the standard form for the hyperboloid of one sheet.



13.6

42.



45. The distance from (x,y,z) to $(-1,0,0)$ is $\sqrt{(x+1)^2+y^2+z^2}$ and the distance from (x,y,z) to the plane $x=1$ is just the distance from (x,y,z) to $(1,y,z)$ (the line connecting these two points is perpendicular to the plane).
So the distance is $x-1$.
Now if the two distances are equal,

$$x-1=\sqrt{(x+1)^2+y^2+z^2} \Rightarrow (x-1)^2=(x+1)^2+y^2+z^2$$

$$\Rightarrow 0 = 4x+y^2+z^2 \Rightarrow x = -\frac{y^2+z^2}{4},$$

an elliptic paraboloid.

15.2

7. Let $y=mx$. Then $\frac{x^2}{x^2+y^2}=\frac{x^2}{x^2+m^2x^2}=\frac{1}{1+m^2}$, and for different values of m, we get different values, so the limit does not exist.

8. Let $y=mx$. Then $\frac{x^2+\sin^2 y}{2x^2+y^2}=\frac{x^2+\sin^2(mx)}{2x^2+m^2x^2}=\frac{1+\frac{\sin^2(mx)}{x^2}}{2+m^2}$.
If we let $m=0$, as $x\to 0$, we get $\frac{1}{2}$. Letting $m=1$, we get $\lim_{x\to 0}\frac{1+\frac{\sin^2 x}{x^2}}{2+1}=\frac{1+1}{3}=\frac{2}{3}$ $\left(\lim_{x\to 0}\frac{\sin x}{x}=1\right)$.
$\frac{1}{2}\neq\frac{2}{3}\Rightarrow$ the limit does not exist.

12. $\lim_{(x,y)\to(0,0)}\frac{x^4-y^4}{x^2+y^2}=\lim_{(x,y)\to(0,0)}\frac{x^2+y^2}{x^2+y^2}(x^2-y^2)=\lim_{(x,y)\to(0,0)}(x^2-y^2)=0.$

13. Let $y=mx^2$. Then $\frac{2x^2y}{x^4+y^2}=\frac{2mx^4}{x^4+(mx^2)^2}=\frac{2m}{1+m^2}$, and taking $m=0$ gives 0 while $m=1$ gives 1. So the limit does not exist.