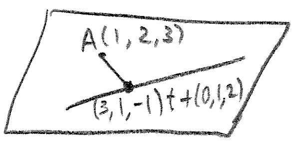
HOMEWORK 3

13.5

26. The plane perpendicular to a line has as its normal vector the line's direction vector: $x=1+t,\ y=2t,\ z=4-3t \Rightarrow \vec{n}=\langle 1,2,-3\rangle$.
So the eq of the plane is $1\cdot(x-x_0)+2(y-y_0)-3(z-z_0)=0$.
$(x_0,y_0,z_0)=(-2,8,10) \Rightarrow (x+2)+2(y-8)-3(z-10)=0$.

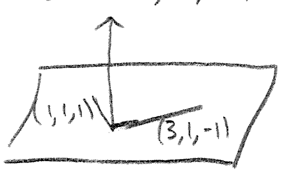
30. Parallel planes have the same normal vector.
So our eq is $2(x-x_0)+4(y-y_0)+8(z-z_0)=0$.
Pick any point in the plane to be (x_0,y_0,z_0): let $t=0$; then
$(x_0,y_0,z_0)=(3,0,8)$
$\Rightarrow\ 2(x-3)+4(y-0)+8(z-8)=0$.

34.



Form a vector using $A=(1,2,3)$ and any point on the line. For convenience, $t=0$ gives $(0,1,2)$.

So $(1,2,3)-(0,1,2)=(1,1,1)$. And the line gives us another vector $(3,1,-1)$.



Taking the cross product, we get the normal vector:

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 3 & 1 & -1 \end{vmatrix} = i\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= -2i+4j-2k$$

$\Rightarrow$ eq is $-2(x-x_0)+4(y-y_0)-2(z-z_0)=0$.

Pick A to be (x_0,y_0,z_0) (we can pick any other point in the plane)

$\Rightarrow\ -2(x-1)+4(y-2)-2(z-3)=0$.

13.5

4. $x = 2t,\ y = 1 - t,\ z = 4 + 3t$

$$\Rightarrow \vec{r}(t) = \langle x(t), y(t), z(t)\rangle = \langle 2t, 1-t, 4+3t\rangle$$
$$= \langle 2,-1,3\rangle t + \langle 0,1,4\rangle$$

So we take the direction vector and add the initial pt (origin)

$$\vec{r}_{new}(t) = \langle 2,-1,3\rangle t + \langle x_0, y_0, z_0\rangle$$
$$= \langle 2,-1,3\rangle t$$

Vector equation: $\vec{r}_{new}(t) = \langle 2,-1,3\rangle t.$

Parametric eq: $x_{new}(t) = 2t,\ y_{new}(t) = -t,\ z_{new} = 3t.$

5. The line perpendicular to a plane is the normal vector, so $\vec{n} = \langle 1,3,1\rangle$ (coefficients of x, y, z).

The equation of the line is then $\vec{r}(t) = \langle 1,3,1\rangle t + \langle x_0, y_0, z_0\rangle$

$$\Rightarrow \vec{r}(t) = \langle 1,3,1\rangle t + \langle 1,0,6\rangle \quad \text{(Vector eq)}$$
$$x = t+1,\ y = 3t,\ z = t+6 \quad \text{(Parametric)}.$$

14. We should check whether the vectors formed by these lines have dot product equal to 0.

$$(2,5,3) - (4,1,-1) = (-2,4,4)$$
$$(5,1,4) - (-3,2,0) = (8,-1,4)$$

Now $(-2,4,4)\cdot(8,-1,4) = -2\cdot 8 + 4\cdot -1 + 4\cdot 4 = -4 \neq 0$, so the two lines are not perpendicular.

22. Rewrite as parametric equations:

$$t = \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1} \Rightarrow x = 2t+1,\ y = 2t+3,\ z = -t+2$$
$$s = \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3} \Rightarrow x = s+2,\ y = -s+6,\ z = 3s-2$$

Not parallel since $\langle 2,2,-1\rangle, \langle 1,-1,3\rangle$ are not multiples of each other.

If they intersect, the x,y,z-coordinates must correspond:

$$\underbrace{2t+1 = s+2,\quad 2t+3 = -s+6}_{\substack{2 = 2s - 4 \Rightarrow s = 3 \\ \Rightarrow t = 2.}},\quad -t+2 = 3s-2$$

But putting this in the 3rd eq $\Rightarrow -2+2 = 3\cdot 3 - 2$ is not true.

So the two lines are skew.

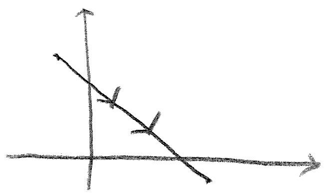
11.1

6. I'll just do part b first.

b) $t = x - 1$, $t = \frac{5-y}{2} \Rightarrow x - 1 = \frac{5-y}{2} \Rightarrow 2x + y = 7$.

And this is valid for $-2 \leq t \leq 3 \Rightarrow -1 \leq x \leq 4$.

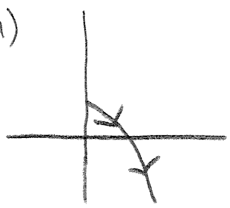
a)



9. The thing to watch out for here is that $x = \sqrt{t} \geq 0$. Keep it in mind.

b) $x = \sqrt{t} \Rightarrow x^2 = t$. $y = 1 - t = 1 - x^2 \Rightarrow y = 1 - x^2$, $x \geq 0$.

a)

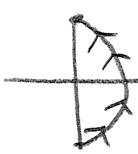


(We do not have the left half of the parabola)

12. a) When you see cos, sin, a good thing to remember is $\cos^2\theta + \sin^2\theta = 1$.

So $\cos\theta = \frac{x}{4}$, $\sin\theta = \frac{y}{5} \Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1$ and $x \geq 0$.

b)



t increasing with arrows

24. The best thing in these is plotting points.

(a) $t = 0 \Rightarrow x = 1$, $y = 0 \Rightarrow$ only III matches.

(b) $t = 0 \Rightarrow x = 0$, $y = 0 \Rightarrow$ I.

(c) $t = 0 \Rightarrow x = 0$, $y = 2 \Rightarrow$ I or IV but we already used I $\Rightarrow$ IV.

(d) $t = 0 \Rightarrow x \approx 1.9$, $y = 2 \Rightarrow$ II.

33. $x = 2\cos\theta$, $y = 2\sin\theta + 1 \Rightarrow x^2 + (y-1)^2 = 4$.

(a) Starting at $(2,1)$: $\theta = 0$. Clockwise - we're going in the reverse direction. One way to do this is to plug $-\theta$ in for θ.

$\Rightarrow x = 2\cos\theta$, $y = -2\sin\theta + 1$, $0 \leq \theta \leq 2\pi$. $(\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta)$.

(b) $x = 2\cos\theta$, $y = 2\sin\theta + 1$, $0 \leq \theta \leq 6\pi$.

(c) $(0,3)$ corresponds to $\frac{\pi}{2}$. Halfway around $\Rightarrow \frac{3\pi}{2}$.

$\Rightarrow x = 2\cos\theta$, $y = 2\sin\theta + 1$, $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$.

14.1

4. $\lim_{t\to 0}\left\langle \frac{e^t-1}{t}, \frac{\sqrt{1+t}-1}{t}, \frac{3}{1+t}\right\rangle = \left\langle \lim_{t\to 0}\frac{e^t-1}{t}, \lim_{t\to 0}\frac{\sqrt{1+t}-1}{t}, \lim_{t\to 0}\frac{3}{1+t}\right\rangle.$

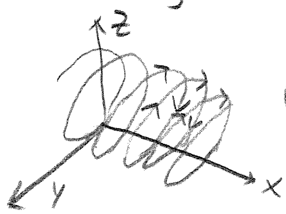
l'Hospital: $\lim_{t\to 0}\frac{e^t-1}{t} = \lim_{t\to 0}\frac{e^t}{1} = 1.$

$\lim_{t\to 0}\frac{\sqrt{1+t}-1}{t} = \lim_{t\to 0}\frac{\frac{1}{2\sqrt{1+t}}}{1} = \frac{1}{2}.$

And $\lim_{t\to 0}\frac{3}{1+t} = 3.$

So $\lim = \langle 1, \frac{1}{2}, 3\rangle.$

9. A circle corresponds to $\langle \cos 2t, \sin 2t\rangle$.
Just adding in the condition $x=t$, we get a spiral formation.



For a better picture, look at 24 $\boxed{VI}$
(It's the same but with the axes changed).

15. Direction vector $\overrightarrow{PQ}$: $(1,2,3)-(0,0,0)=(1,2,3)$.
$\vec{r}(t) = \langle 1,2,3\rangle t + \langle x_0, y_0, z_0\rangle = \langle 1,2,3\rangle t$ (Plug in $P=(0,0,0)$).
Parametric: $x=t$, $y=2t$, $z=3t$.

19. This is the spiral (see exercise 9) - $\boxed{VI}$.

20. As $t\to\infty$, $x\to\infty$, $y\to\infty$, $z\to 0$. - $\boxed{II}$

21. As $t\to\infty$, $y\to 0$ and $x=t$, $z=t^2 \Rightarrow z=x^2$, so as $t\to\infty$, the graph looks more and more like the parabola $z=x^2 \Rightarrow \boxed{IV}$.

22. This is spiral-like, but now we have multiplying factors in x, y also. You can also observe that $t\to\infty \Rightarrow x=0, y=0, z=0 \Rightarrow \boxed{I}$.

23. This is the only one that is periodic $\Rightarrow \boxed{V}$.

24. As $t\to 0$ from above, $z\to -\infty \Rightarrow \boxed{III}$.

14.1

34. Parametrizing the circle $x^2+y^2=4$, we get $x=2\cos\theta$, $y=2\sin\theta$.
The resulting function also must satisfy $z=xy$,
so $z=(2\cos\theta)(2\sin\theta)=4\sin\theta\cos\theta$.
Then $\vec{r}(\theta)=\langle 2\cos\theta, 2\sin\theta, 4\sin\theta\cos\theta\rangle \quad 0\le\theta\le 2\pi$
$= \langle 2\cos\theta, 2\sin\theta, 2\sin 2\theta\rangle$ if you prefer.

36. $y=x^2$ can be parametrized as $x=t$, $y=t^2$.
Then $z=4t^2+(t^2)^2=4t^2+t^4$.
So $\vec{r}(t)=\langle t, t^2, 4t^2+t^4\rangle$.