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Garnett

MATH 32A - PRACTICE FINAL EXAM

March 8, 2006.

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Name:(Last, First)\_\_\_\_\_

Signature \_\_\_\_\_

Quiz Section (or Time and TA's Name) \_\_\_\_\_

Answers on exam. No books or notes. You may remove the scratch paper at the end of your exam. ( $n$ ) means the problem is worth  $n$  points.

(15) 1. Let

$$\vec{a} = \vec{j} - \vec{k}$$

and

$$\vec{b} = 2\vec{i} + \vec{j} + 2\vec{k}.$$

Find a vector  $\vec{u}$  such that

(i)  $\vec{u} = c\vec{b}$  for some scalar  $c$ , and

(ii)  $\vec{a} - \vec{u} \perp \vec{b}$ ,

where  $\perp$  means orthogonal or perpendicular.

Draw a picture illustrating the relations between  $\vec{a}$ ,  $\vec{b}$  and  $\vec{u}$ .

(15) 2. Suppose that  $\vec{u} \cdot (\vec{v} \times \vec{w}) = 2$ . Find the following:

(a)  $(\vec{u} \times \vec{v}) \cdot \vec{w}$ .

(b)  $\vec{u} \cdot (\vec{w} \times \vec{v})$ .

(c)  $\vec{v} \cdot (\vec{u} \times \vec{w})$ .

(d)  $(\vec{u} \times \vec{v}) \cdot \vec{v}$ .

(15) 3. Find the length of the curve

$$\vec{r}(t) = (\sin 2t, 2t^{\frac{3}{2}}, \cos 2t)$$

for  $0 \leq t \leq 1$ .

(10) 4. Suppose a particle in 3-space moves with motion  $\vec{r}(t)$ , velocity  $\vec{v}(t)$  and acceleration  $\vec{a}(t)$ . Assume that the speed  $|\vec{v}(t)|$  is constant. Prove that

$$\vec{v}(t) \cdot \vec{a}(t) = 0.$$

(35) 5. Let  $\vec{r}(t) = t\vec{i} + \frac{t^2}{2}\vec{j} + \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}\vec{k}$ , for  $1 \leq t \leq 4$ .

(a) Find the velocity vector  $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$  and the speed  $\frac{ds}{dt} = |\vec{v}(t)|$ .

(b) Find the the unit tangent vector  $T(t)$ .

(c) Find the derivative  $T'(t)$  and its length  $|T'(t)|$ . Calculate carefully here, you will need these results below.

(d) Find the curvature  $\kappa(t)$ .

(e) Find the normal vector  $N(t)$ .

(f) For  $t = 1$  find the binormal vector  $B(1)$ .

(g) For  $t = 1$  find the equation of the osculating plane for the curve  $\vec{r}(t)$ .

(20) 6. Let

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}.$$

(a) Find the partial derivatives  $f_x(x, y, z)$ ,  $f_y(x, y, z)$  and  $f_z(x, y, z)$ .

(b) Find the linear approximation of  $f(x, y, z)$  at the point  $(3, 2, 6)$ .

(c) Use your answer to (b) to approximate  $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$ .

(25) 7. Find the equation of the tangent plane and normal line to the surface

$$xy = \ln(x + z)$$

at the point  $(1, 0, 0)$ .

(25) 7'. (a) Give the  $\epsilon - \delta$  definition of

$$\lim_{(x,y) \rightarrow (a,b)} f(x,y) = L.$$

(b) Let  $f(x,y) = x^2 + y^2 - 1$ . Prove using the definition in (a) that

$$\lim_{(x,y) \rightarrow (1,0)} f(x,y) = 0.$$

(20) 8. The temperature at a point  $(x, y)$  is  $T(x, y)$ , measured in degrees Celsius. A bug crawls so that its position after  $t$  seconds is given by

$$(x(t), y(t)) = (\sqrt{1+t}, 2 + \frac{t}{3})$$

where  $x$  and  $y$  are measured in centimeters. The temperature function  $T(x, y)$  satisfies  $T_x(2, 3) = 4$  and  $T_y(2, 3) = 3$ . How fast is the temperature rising on the bug's path after  $t = 3$  seconds?

(20) 8'. Near a buoy, the depth of a lake at the point  $(x, y)$  is  $z = 200 = 0.02x^2 - 0.001y^3$ , where  $x, y$  and  $z$  are measured in meters. A boat starts at  $(80, 60)$  and moves in a straight line toward the buoy, which is at  $(0, 0)$ . Is the water under the boat getting deeper or shallower as the boat starts off? Calculate an explicit directional derivative to justify your answer.

(25) 9. On the closed triangle in the  $xy$ -plane with vertices  $(0, 0)$ ,  $(6, 0)$  and  $(0, 6)$  find the absolute maximum and minimum values of the function

$$f(x, y) = x^3y + x^2y^2 - 4x^2y.$$

(20) 10. The plane  $x + y + 2z = 2$  intersects the paraboloid  $z = x^2 + y^2$  in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.