MATH 252A - Fall 2011 - Topics in Complex Analysis

Time and Place: MS 6221, MWF 10:00, starting Sept. 23.

Office hours: John Garnett MWF 2:00 in MS 7941.

Texts:

1. *Harmonic Measure*, by Garnett and Marshall, (Cambridge University Press, 2008, cheaper in paperback).

2. Harmonic Measure, Geometric and Analytic Points of View, by Capogna, Kenig and Lanzani, (American Mathematical Society, 2005, also \$28 for members).

Plan:

We will discuss harmonic measure in simply connected domains in the plane in and in $\mathbf{R}^d, d \geq 3$. The two theories are somewhat different and use different methods; univalent function theory in the case of two dimensions, and PDE and Geometric Measure Theory in the case of higher dimensions. We will spend about 1/2 of the course on each case, and use the well developed planar theory as a road map for what people should try to prove in higher dimensions.

There will be some homework problems to keep students alert. This course will be very similar to Math 285G of Spring 2009 and students who completely mastered 285G from Spring 2009 and are still at UCLA should not take 252B.

Topics:

- 1. Brownian Motion and Harmonic Measure.
- 2. Simply connected plane domains, Koebe, F. and M. Riesz, etc.
- 3. $\log \varphi'$, Bloch functions, quasicircles, chord-arc curves, etc.
- 4. Makarov's theorems on Hausdorff measure.
- 5. Potential Theory for domains in $\mathbf{R}^d, d \geq 3$.
- 6. Dahlberg's Theorem
- 7. Reifenberg flat domains and chord-arc domains.

The first four topics come from the Garnett-Marshall book, topics 5 and 7 come from Capogna-Kenig-Lanzani, and topic 6 is from a paper of Jerison and Kenig.

- J. Garnett