

UCLA NUMBER THEORY LEARNING SEMINAR, SPRING 2023

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1. SUGGESTED REFERENCES

The primary reference for this seminar will be [Hong's p-adic Hodge theory notes](#).

2. ORDER OF TALKS

- (1) **Finite flat group schemes, Frobenius.** - Recollections on group schemes, in particular algebraic groups and elliptic curves, basic theory, and Frobenii. This should cover roughly section 1 of Hong.
- (2) **p -divisible groups.** - This talk will introduce p -divisible groups, basic properties, and connections to Dieudonne modules, following section 2, chp 2 of Hong.
- (3) **Hodge–Tate Decomposition I.** - The first of two talks introducing the Hodge-Tate decomposition, motivated from complex geometry, following roughly Hong II.3.1-II.3.3, focused on introducing formal groups and the p -adic logarithm
- (4) **Hodge–Tate Decomposition II.** - Continuing last weeks talk, this week we will finish section II.3 of Hong's notes, discussing the Hodge–Tate decomposition for Tate modules, and generic fibers of p -divisible groups.
- (5) **Fontaine's Formalism and Period Rings.** - This talk introduces period rings, Fontaine's resolution to Grothendieck's conjecture about the Grothendieck mysterious functor, following section III.1 of Hong.
- (6) **de Rham representations.** - Following Hong III.2, we introduce de Rham representations, the first kind of representations related to period rings which we will be interested in.
- (7) **Crystalline representations.** This week, we discuss so-called crystalline representations and finish the discussion of Grothendieck's conjecture, roughly following Hong III.3.
- (8) **Geometric structure of the Fargues–Fontaine curve.** We turn our attention to the Fargues-Fontaine curve again, discussing its geometric properties as in Hong IV.2
- (9) **Vector bundles on the Fargues–Fontaine curve I:** . Following Hong IV.3.1-IV.3.4, we introduce the theory of vector bundles on the Fargues-Fontaine curve, one of the main tools in modern p -adic Hodge theory.
- (10) **Vector bundles on the Fargues–Fontaine curve II: Applications.** In the final talk, we conclude the discussion on the properties of vector bundles on the Fargues-Fontaine curve, and connect them back to p -adic representations to give various applications, following Hong IV.3.5-IV.4.2.