UCLA NUMBER THEORY LEARNING SEMINAR 22X

ETALE COHOMOLOGY AND THE WEIL CONJECTURE

We roughly follow the contents of Freitag and Kiehl (FK). Also useful are the Stacks Project, Milne's Lecture notes on Etale Cohomology (and possibly the book as well), and the notes from Brian Conrad's 2016-2017 seminar on etale cohomology.

1. Basics of Etale Theory

1.1. **Etale morphisms, algebraically and classically.**

1.2. **Etale morphisms, formally.**

1.3. **Etale Fundamental Groups.**

1.4. **Grothendieck topologies and sites.**

1.5. **More on Etale Sheaves, the Kummer Sequence.**

2. First Steps in Etale Cohomology

2.1. **Calculations for Curves.** Compute some cohomology of curves.

2.2. **Constructible Sheaves and Finiteness Theorems (1.5 talks).** See Stacks Project 59.71, Freitag–Kiehl I.4. Basic properties. Recovering torsion sheaves as colimits of constructible ones. Relationship with finite morphisms, compatibility with pullback and pushforward. Cohomology of torsion sheaves. Extension of a sheaf by 0 (See the first couple subsections of FK I.8) and any other sheaf constructions we might need later.

2.3. **Proper Base Change (1 talk).** See Stacks Project for full proofs. See FK I.6. The goal is to prove, or at least give a sketch, for the Proper Base Change Theorem (Theorem 6.1 in FK).

2.4. **Smooth Base Change (1 talk).** See Stacks Project for full proofs. See FK I.7. The goal is to prove, or at least give a sketch, for the Smooth Base Change Theorem (Theorem 7.3 in FK).

2.5. **Adic Formalism, ℓ-adic Sheaves (2 talks).** See FK I.12 (especially Theorem 12.15, the "main theorem" and the "only nontrivial theorem" of the section according to FK), Conrad, or Milne (in that order). Or read SGA. Review the Mittag–Leffler condition. Artin–Rees categories, adic systems. The goal here is to get finiteness results about ℓ-adic cohomology and to make sense of derived functors of direct image and extension by zero. Brian Lawrence wrote some notes for the Conrad seminar that will probably be very useful.

2.6. **Duality theorems (2 talks).** FK II.1. Poincare (and Verdier) duality. Also discuss compactly supported cohomology, and connect to the Weil conjectures.

2.7. **Comparison theorems (1 talk, optional?)** FK I.11. The Artin–Grothendieck Theorem (over $\mathbb{C}$). How does etale cohomology compare to singular cohomology? See Milne §21. We might be able to skip this, or incorporate into talks on adic stuff (i.e. adic comparison theorems).

3. Proof of the Weil Conjectures

Most of the following is from Freitag and Kiehl.

3.1. **Frobenius, Grothendieck–Lefschetz trace formula, Purity results (1.5 talks).** See Conrad notes.

3.2. **Grothendieck's Formula for L-series (1 talk).** See FK II.4.

3.3. **Basics Lefschetz pencils and their singularities (1 talk).** FK III.1-2. Existence, classification of singularities/double points.

3.4. **Monodromy of Lefschetz pencils (3 talks).** FK III.3-6. Formalism, Picard–Lefschetz formula, behavior under base change, computations.

3.5. **Global monodromy of Lefschetz pencils (1 talk).** FK III.7. Get to Kazhdan–Margulis (Theorem 7.5 in FK).

3.6. **Deligne's Proof of the Weil Conjectures (1.5 talks).** FK IV. If time permits, we can discuss applications and generalizations.