UCLA PARTICIPATING NUMBER THEORY SEMINAR WINTER 2023

LOGAN HYSLOP AND JACOB SWENBERG

This is a syllabus/rough outline for the UCLA Winter 2023 Math 290B, Participating Seminar: Current Literature in Number Theory. The topic will be on p-adic geometry, primarily following the Berkeley Lectures by Scholze and Weinstein. When we say "lecture n," it is implicitly assumed to mean in the Berkeley lectures, unless otherwise stated.

Talks will be held Tuesdays, 2-3 pm in Math Sciences room 7608.

1. Lineup of Talks

1. Adic Spaces and Pre-adic Spaces. This talk is to introduce the language of adic spaces, discussing Huber rings, Tate rings, continuous valuations, affinoid spaces and their rational subsets, and adic spaces. If there is time, discuss the failure of some Huber pairs to not be sheafy, and discuss pre-adic spaces as a fix (Lectures 2 and 3).

2. Examples of Adic Spaces and Complements on them. We will continue the discussion of adic spaces, starting off with examples (lecture 4) of adic spaces, then discuss adic morphisms, analytic points, and analytic adic spaces (lecture 5).

3. Perfectoid Rings and Spaces. This talk should introduce perfectoid rings, specifically about definitions and examples, tilting, perfectoid spaces, and almost mathematics (lectures 6 and 7).

4. Diamonds, Day 1. Introduce the pro-étale site, (lecture 8, see also this paper by Bhatt-Scholze), the definition for diamonds, complements on the pro-étale topology, and quasi-pro-étale morphisms (lecture 9).

5. Diamonds, Day 2. This week, we discuss *G*-torsors, untilting, diamonds associated to adic spaces, and the underlying space of a diamond (lectures 9 and 10).

6. Fargues-Fontaine. Here, we will introduce the Fargues-Fontaine curve as a diamond, the infinitesimal period ring and the de Rham period ring. (See for example Mathew Morrow's paper).

7. Diamonds, Day 3. This talk will be about more examples of diamonds, Banach-Colmez spaces (lecture 15), and (if there is time), Drinfeld's lemma for diamonds (lecture 16).

8. The v-topology and v-sheaves. This week, we discuss the v-topology, small v-sheaves, special v-sheaves and morphisms of v-sheaves (lecture 17). Also, if time, sketch the association of v-sheaves to perfect schemes and formal schemes (lecture 18).

9. B_{dR}^+ -affine Grassmannians. We discuss the B_{dR}^+ -affine Grassmannian, the definition of it, Schubert varieties, the Demazure resolution, and \mathcal{G} -torsors (lecture 19).

<u>10. More on affine Grassmannians.</u> This week, we introduce families of affine Grassmannians (lecture 20), and if there is time, affine flag varieties (lecture 21).

2. Some Resources

1. The Berkeley Lectures on p-adic geometry, by Peter Scholze and Jared Weinstein.

2. Notes from the 2017 Arizona Winter School by Bhargav Bhatt, Ana Caraiani, Kiran S. Kedlaya, and Jared Weinstein.

3. A brief introduction to Adic Spaces by Brian Conrad.

4. Adic Spaces by Sophie Morel.

5. The Fargues-Fontaine Curve and Diamonds by Mathew Morrow.

6. The pro-étale topology for schemes by Bhargav Bhatt and Peter Scholze. 7. Perfectoid Spaces by Peter Scholze.