7. $\{x_1 + 2x_2 + x_3 = 0\} = V$

find $\mathcal{B}$ basis

s.t.

$$\underbrace{\left[\underbrace{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}}_{\text{in place}}\right]_{\mathcal{B}}} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$1 - 2 + 1 = 0$

$\mathcal{B} = (\vec{v_1}, \vec{v_2})$

$\underline{\vec{v_1}}, \underline{\vec{v_2}}$ in V lin indep, span

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 2\vec{v_1} - \vec{v_2}$$

$$\underbrace{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} - 2\vec{v_1}} = \underbrace{-\vec{v_2}}$$

$$\begin{pmatrix}1\\-1\\1\end{pmatrix} = 2\vec{v_1} - \vec{v_2}$$

← the plane

<u>if</u> we find $\vec{v_1}$ in V

<u>then</u> $\vec{v_2}$ forced to be $\underbrace{2\vec{v_1} - \begin{pmatrix}1\\-1\\1\end{pmatrix}}_{\text{in } V}$

vice versa

$$\vec{v_1} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$$\vec{v_2} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$$

pick $\vec{v_1}$ in V

s.t. $2\vec{v_1} - \begin{pmatrix}1\\-1\\1\end{pmatrix}$ lin indep from $\vec{v_1}$

"Lin. independence is generic"

det $\neq 0$ det $= 0$

$\vec{V_1}$ <u>not</u> in span $\begin{pmatrix}1\\-1\\1\end{pmatrix}$

$$2\vec{v_1} - \begin{pmatrix}1\\-1\\1\end{pmatrix} = c\vec{v_1} \iff (2-c)\vec{v_1} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$$

pick any $\vec{v_1}$ in V

which is not in span $\begin{pmatrix}1\\1\\1\end{pmatrix}$

solve for $\vec{v_2}$

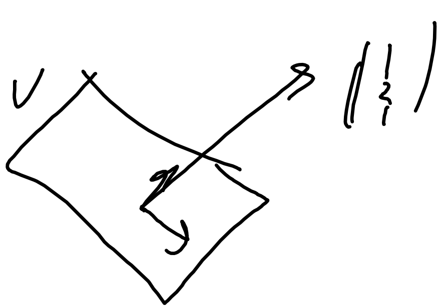
Justify this is a basis

How to find elts in V

$x_1 + 2x_2 + x_3 = 0$

$\Updownarrow$

$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \perp \begin{pmatrix}1\\2\\1\end{pmatrix}$



$x_1 + 2x_2 + x_3 = 0$

underdetermined

∞ly solutions

set of solutions is $\boxed{2d}$

$x_1 = 0$

$2x_2 + x_3 = 0$

~~$x_2 = 0 \Rightarrow x_3 = 0$~~

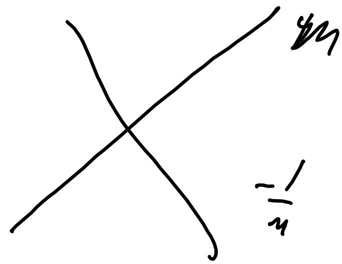
* $x_2 = 1$, $2 + x_3 = 0 \Rightarrow x_3 = -2$

$$\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \longmapsto \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$x_2 = 0$ $x_1 = c$

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$ $\Rightarrow x_3 = -1$

$\begin{pmatrix} x \\ y \end{pmatrix} \perp \begin{pmatrix} -y \\ x \end{pmatrix}$



$\vec{v}, \vec{w}$

$\vec{v} \times \vec{w} \perp \vec{v}$ and $\vec{w}$

$$\begin{array}{cccccc} x & y & z & w & & \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} y + 2w = 0 \\ z + 4w = 0 \end{cases}$$

$y = -2w, \quad z = -4w$

pick x

pick w

$$\left\{ \begin{pmatrix} x \\ -2w \\ -4w \\ w \end{pmatrix} \right\}$$

7.4.22

$$\boxed{\lim_{t\to\infty} A^t}$$

$$A = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$$

$$\lim_{t\to\infty} A^t$$

$$A^t = \begin{pmatrix} f_{11}(t) & f_{12}(t) \\ f_{21}(t) & f_{22}(t) \end{pmatrix}$$

$$\lim_{t\to\infty} A^t = \begin{pmatrix} \lim_{t\to\infty} f_{11}(t) & \lim_{t\to\infty} f_{12}(t) \\ \lim_{t\to\infty} f_{22}(t) & \lim_{t\to\infty} f_{22}(t) \end{pmatrix}$$

formulae $f_{ij}(t)$?

$$D = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

$$D^t = \begin{pmatrix} a^t & 0 \\ 0 & b^t \end{pmatrix}$$

$$S^{-1} A S = D$$

$$A = S D S^{-1}$$

$$A^t = \underbrace{S D \overbrace{S^{-1} S}^{I} D \overbrace{S^{-1} \ldots S}^{I} D S^{-1}}_{t}$$

$$= S D^t S^{-1}$$

$$\det(A - \lambda I) = \det\begin{pmatrix} 0.8-\lambda & 0.6 \\ 0.2 & 0.4-\lambda \end{pmatrix}$$

$$= (0.8-\lambda)(0.4-\lambda) - 0.6 \cdot 0.2$$

$$= 0.32 - 0.8\lambda - 0.4\lambda + \lambda^2 - 0.12$$

$$= \lambda^3 - 1.2\lambda + 0.2$$

$$= (\lambda - 1)(\lambda - 0.2)$$

g.m $\leq$ a.m

λ	a.m.	g.m
1	1	1
0.2	1	1

$\boxed{\text{diagonalizable}}$

$gm(\lambda), \quad am(\lambda)$

$\underline{gm(\lambda) \leq am(\lambda)}$

$$\sum_{\lambda \text{ eigenvalue}} am(\lambda) = n$$

diagonalizability $\Longleftrightarrow$ $\boxed{\sum_{\lambda \text{ eival}} gm(\lambda) = n}$

$\underbrace{\dim \underline{\ker(A - \lambda I)}} = gm(\lambda)$

$$\sum_{\lambda \text{ eival}} gm(\lambda) = n \Longleftrightarrow gm(\lambda) = am(\lambda) \text{ for all } \lambda$$

if $\underbrace{gm(\lambda)} < \underbrace{am(\lambda)}$

$\underbrace{O^T W O} = D$

$D^T = D$

$(O^T W O)^T = O^T W^T O$

$$(\phi^T W \phi)^T = D^T$$
$$= D$$
$$\| $$
$$\phi^T W^T \phi = \phi^T W \phi$$

$$W^T = W$$

$$S^{-1} A S = \begin{pmatrix} 1 & 0 \\ 0 & 0.2 \end{pmatrix}$$

$$A = S \begin{pmatrix} 1 & 0 \\ 0 & 0.2 \end{pmatrix} S^{-1}$$

$$A^t = S \begin{pmatrix} 1 & 0 \\ 0 & 0.2 \end{pmatrix}^t S^{-1}$$

$$= S \begin{pmatrix} 1 & 0 \\ 0 & 0.2^t \end{pmatrix} S^{-1}$$

$$\boxed{\lim_{t \to \infty} A^t} = \boxed{S \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} S^{-1}}$$

$$\lim_{t \to \infty} 0.2^t = 0$$

$$A = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$$

$$A - I = \begin{pmatrix} -0.2 & 0.6 \\ 0.2 & -0.6 \end{pmatrix}$$

$$-0.2x + 0.6y = 0$$

$$x = 0.6 \quad y = 0.2$$

$$\begin{pmatrix} -0.2 \\ 0.6 \end{pmatrix} \perp \begin{pmatrix} 0.6 \\ 0.2 \end{pmatrix}$$

$$A \begin{pmatrix} 0.6 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.2 \end{pmatrix}$$

$$A \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$\underline{A - 0.2I}$

$$= \begin{pmatrix} 0.6 & 0.6 \\ 0.2 & 0.2 \end{pmatrix}$$

$$\ker(A - 0.2I) = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$$

$$S = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$S^{-1} = -\frac{1}{4} \begin{pmatrix} -1 & -1 \\ -1 & 3 \end{pmatrix}$$

$$\lim_{t \to \infty} A^t = S \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} S^{-1}$$

$$= -\frac{1}{4} \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -1 & 3 \end{pmatrix}$$

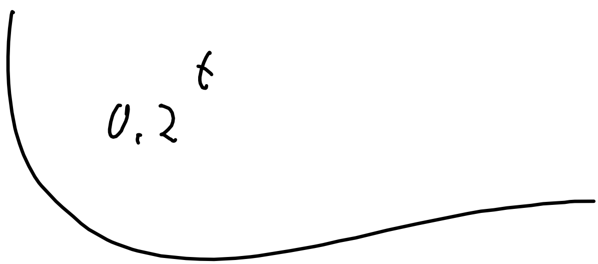
$$= -\frac{1}{4} \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix} = -\frac{1}{4} \begin{pmatrix} -3 & -3 \\ -1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3/4 & 3/4 \\ \underline{1/4} & \underline{1/4} \end{pmatrix}$$

$$A^t \left(a \begin{pmatrix} ? \\ 1 \end{pmatrix} + b \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)$$

$$= a \begin{pmatrix} ? \\ 1 \end{pmatrix} + b \cdot \underbrace{0.2^t \begin{pmatrix} 1 \\ -1 \end{pmatrix}}_{\text{really small}}$$

really fast



$\boxed{18, 1, 42}$

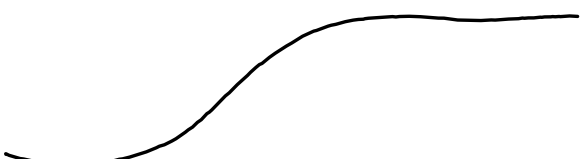
find 2×2 symmetric matrix B

so that $B^3 = \underbrace{\frac{1}{5}\begin{pmatrix} 12 & 14 \\ 14 & 33 \end{pmatrix}}_{A}$

I give A you give B

$\underline{B^3 = A}$ → you win

$f(x) = \sqrt[3]{x}$



$\begin{pmatrix} a & b \\ b & d \end{pmatrix}^3 = \ldots$

4 eqs

3 unknowns

degree 3

$A \rightsquigarrow B$ s.t. $B^3 = A$

$$D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

find B s.t $B^3 = D$ in this case?

$$B = \begin{pmatrix} \lambda_1^{1/3} & 0 \\ 0 & \lambda_2^{1/3} \end{pmatrix}$$

$$B^3 = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$$A = \frac{1}{5}\begin{pmatrix} 12 & 14 \\ 14 & 33 \end{pmatrix}$$

$$A = O D O^T$$

O orthog

D diagon

$$D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$$C = \begin{pmatrix} \lambda_1^{1/3} & 0 \\ 0 & \lambda_2^{1/3} \end{pmatrix}$$

$$C^3 = D$$

want B s.t. $B^3 = A$

$S^{-1}AS = D$, cols S are eigenvectors v_i

$$\begin{tikzcd} e_i \arrow[r, mapsto, "S"] & v_i \arrow[r, mapsto, "A"] & \lambda_i v_i \arrow[d, mapsto, "S^{-1}"] \\ & & \lambda_i S^{-1} v_i = \lambda_i e_i \end{tikzcd}$$

$S^{-1}AS : e_i \mapsto \lambda_i e_i$

i^{th} col of $S^{-1}AS = \lambda_i e_i$

$$S^{-1}AS = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

$$\boxed{S^{-1}AS = D \quad *** \quad \xrightarrow{\text{so}} AS = SD \qquad A = SDS^{-1}}$$

A_{sym} want to find B_{sym} $B^3 = A$

O orthog $\iff O^T = O^{-1}$

$A = ODO^T$ $\quad D$ diagonal

$\downarrow$ spectral thm

$= ODO^{-1}$

$$D = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix}$$

$$C = \begin{pmatrix} \lambda_1^{1/3} & \\ & \lambda_2^{1/3} \end{pmatrix}$$

$$C^3 = D$$

$B = OCO^T$, B is symmetric

$$B^3 = OC\overbrace{O^TO}^{I}C\overbrace{O^TO}^{I}CO^T$$
$$= OC^3O^T$$
$$= ODO^T$$
$$= A$$

$$B^T = OC^TO^T$$
$$= OCO^T$$
$$= B$$

$A = ODO^T$

$= O C^3 O^T$

$= (O C O^T)^3$

$\therefore \; B \overset{\text{def}}{=} \underline{OCO^T}$

find $\underline{O}$

find $\underline{C} \longrightarrow \begin{pmatrix} \lambda_1^{1/3} & 0 \\ 0 & \lambda_2^{1/3} \end{pmatrix}$

λ_1, λ_2

compute orthogonal diagonalization

$\underline{A}$, find B s.t. $B^3 = A$

never use the word symmetry

if A diagonalizable $\quad S^{-1}AS = D$

$A = SDS^{-1}$

$C^3 = D$, C diagonal

$\underline{B} = SCS^{-1}$

$$\boxed{A = \frac{1}{5}\begin{pmatrix} 12 & 14 \\ 14 & 37 \end{pmatrix}}$$

orthonormally diagonalize

- char poly
- roots → eivs

  compute basis eigenspace $\ker(A - \lambda I)$, using RREF

* make basis ONB, Gram Schmidt

- concatenate
- (put them together)

$\boxed{8.1.27}$

diagonalize

$$\begin{pmatrix} 1 & & 0 & & 1 \\ & 1 & & \iddots & \\ 0 & & \ddots & & 0 \\ & \iddots & & & \\ 1 & & 0 & & 1 \end{pmatrix}$$

$n=2$ $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$n=3$ $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$n=4$ $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$

$n=5$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$

$$A_n = \begin{pmatrix} 1 & & & & 1 \\ & \ddots & & \iddots & \\ & & \times & & \\ & \iddots & & \ddots & \\ 1 & & & & 1 \end{pmatrix} \quad n \times n$$

$$\det(A_n - \lambda I) = \det \begin{pmatrix} 1-\lambda & & & & 1 \\ & 1-\lambda & & \iddots & \\ & & \ddots & & \\ & \iddots & & \ddots & \\ 1 & & & & 1-\lambda \end{pmatrix}$$

$$A_n \vec{x} = \lambda \vec{x} \qquad \vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\begin{pmatrix} x_1 + x_n \\ x_2 + x_{n-1} \\ \vdots \\ x_1 + x_n \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$x_1 + x_n = \lambda x_1$$

$$x_2 + x_{n-1} = \lambda x_2$$

$$x_3 + x_{n-2} = \lambda x_3$$

$$x_4 + x_{n-3} = \lambda x_4$$

$$\vdots$$

$$x_1 + x_n = \lambda x_n$$

$$\vec{x} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$A\vec{x} = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$A - 2I = \begin{pmatrix} -1 & & & 1 \\ & \ddots & \iddots & \\ & \iddots & \ddots & \\ 1 & & & -1 \end{pmatrix}$$

$\ker(A - 2I)$

$\ker(A-2I)$

$n=2$ $\quad A=\begin{pmatrix}1&1\\1&1\end{pmatrix}$ $\quad A-2I=\begin{pmatrix}-1&1\\1&-1\end{pmatrix}$

null $=1$

$\underline{1\text{ dim}}$

$n=3$ $\quad A=\begin{pmatrix}1&0&1\\0&1&0\\1&0&1\end{pmatrix}$, $\quad A-2I=\begin{pmatrix}-1&0&1\\0&-1&0\\1&0&-1\end{pmatrix}$

rk $=2$

null $=1$

$\underline{1\text{ dim}}$

$n=4$ $\quad A=\begin{pmatrix}1&&&1\\&1&1&\\&1&1&\\1&&&1\end{pmatrix}$, $\quad A-2I=\begin{pmatrix}-1&&&1\\&-1&1&\\&1&-1&\\1&&&-1\end{pmatrix}$

rk $=2$

$\underline{\text{null}=2}$

$\underline{2\text{ dim}}$

$n=5$ $\quad A=\begin{pmatrix}1&&&&1\\&1&&1&\\&&1&&\\&1&&1&\\1&&&&1\end{pmatrix}$, $\quad A-2I\begin{pmatrix}-1&&&&1\\&-1&&1&\\&&-1&&\\&1&&-1&\\1&&&&-1\end{pmatrix}$ $\quad$ dim $=2$

rk $=3$

null $=2$

$n=6$

$$A = \begin{pmatrix} 1 & & & & & 1 \\ & 1 & & & 1 & \\ & & 1 & 1 & & \\ & & 1 & 1 & & \\ & 1 & & & 1 & \\ 1 & & & & & 1 \end{pmatrix}$$

$$A-2I = \begin{pmatrix} -1 & & & & & 1 \\ & -1 & & & 1 & \\ & & -1 & 1 & & \\ & & 1 & -1 & & \\ & 1 & & & -1 & \\ 1 & & & & & -1 \end{pmatrix}$$

$rk = 3$

$null = 3$

$\boxed{\text{Conjecture}}$

n even: 2-eigenspace has dim $= \frac{n}{2}$

n odd: 2-eigenspace has dim $= \frac{n-1}{2}$

$$\begin{pmatrix} -1 & & & 1 \\ & -1 & 1 & \\ & 1 & -1 & \\ 1 & & & -1 \end{pmatrix} \underset{R_3 \mapsto R_2+R_3}{\overset{R_4 \mapsto R_1+R_4}{\sim}} \begin{pmatrix} -1 & & & 1 \\ & -1 & 1 & \end{pmatrix} \sim \begin{pmatrix} 1 & & & -1 \\ & 1 & -1 & \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ -1 \end{pmatrix}$$

$x_1 = 1$ $\quad x_i = 0 \quad 2 \le i \le n-1$

$x_n = -1$

$$A \underline{\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ -1 \end{pmatrix}} = \begin{pmatrix} 1-1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1-1 \end{pmatrix} = \vec{0}$$

0 - eigenvalue

ker(A)

$$\begin{pmatrix} 1 & & & & 1 \\ & 1 & & 1 & \\ & & 1 & & \\ & 1 & & 1 & \\ 1 & & & & 1 \end{pmatrix} \underset{R_3 - R_2}{\overset{R_4 - R_1}{\sim}} \begin{pmatrix} 1 & & & & 1 \\ & 1 & & 1 & \\ & & 1 & & \\ & & & & \\ & & & & \end{pmatrix}$$

(c,1). dim 0-eigenspace is $\frac{n}{2}$ n even

" " " " $\frac{n-1}{2}$ n odd

n even: $gm(2) = \frac{n}{3}$

$gm(0) = \frac{n}{2}$

$\frac{n}{2} + \frac{n}{2} = n$

$$A_n \sim \begin{pmatrix} 2 & & & & \\ & \ddots & & & \\ & & 2 \; 0 & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix}$$

n odd: $gm(2) = \frac{n-1}{2}$

$gm(0) = \frac{n-1}{2}$

$\frac{n-1}{2} + \frac{n-1}{2} = n-1$

$n=3$. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$gm(1) = 1$

n odd

$-R_{\frac{n-1}{2}}$

$\vdots$

$-R_2$

$-R_1$

$\frac{n+1}{2}$ th column

$\sim$

n odd

$$A_n e_{\frac{n+1}{2}} = e_{\frac{n+1}{2}}$$

$e_1 \cdots e_{\frac{n+1}{2}} \cdots e_n$

midpoint

gm(1) $= 1$ for n odd

$e_{\frac{n+1}{2}}$

$A_n - I =$  $\sim$ 

rk $n-1$

n even

$\underline{gm(2)} = \frac{n}{2}$

$\underline{gm(0)} = \frac{n}{2}$

$$\begin{pmatrix} 2 & & & & & \\ & \ddots & & & & \\ & & 2 & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}$$

$\underbrace{\qquad}_{\frac{n}{2}} \ \underbrace{\qquad}_{\frac{n}{2}}$

n odd

$$\left.\begin{array}{l} \underline{gm(2)} = \frac{n-1}{3} \\ \underline{gm(0)} = \frac{n-1}{3} \end{array}\right\} + \quad n-1$$

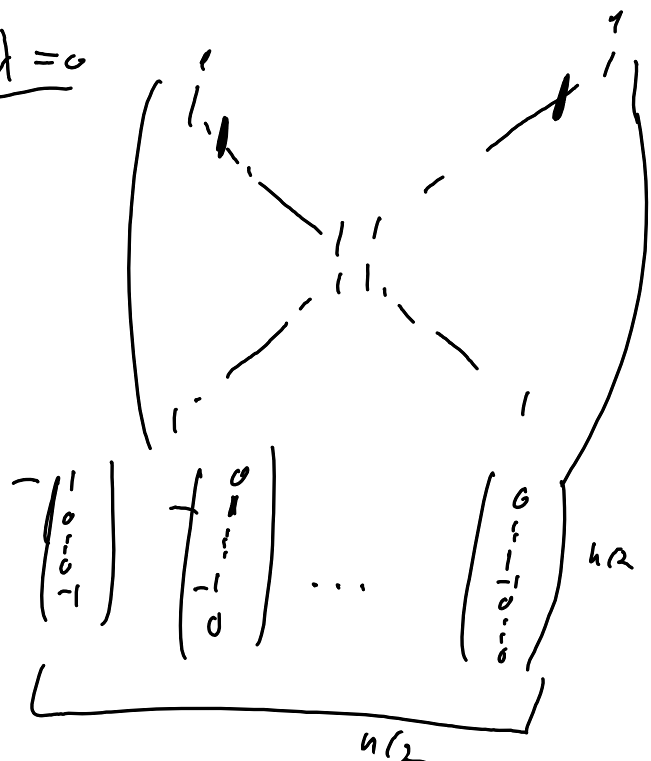
$gm(1) = 1$

$$\begin{pmatrix} 2 & & & & & & \\ & \ddots & & & & & \\ & & 2 & & & & \\ & & & 1 & & & \\ & & & & 0 & & \\ & & & & & \ddots & \\ & & & & & & 0 \end{pmatrix}$$

$\underbrace{\qquad}_{\frac{n-1}{2}} \ \underbrace{\ }_{1} \ \underbrace{\qquad}_{\frac{n-1}{2}}$

	n even	n odd
$\lambda=3$		—
$\lambda=0$		—

n even, $\lambda=0$



$x_1 + x_n = 0$

$x_2 + x_{n-1} = 0$

$\vdots$

$x_{\frac{n}{2}} + x_{\frac{n}{2}+1} = 0$

$$A = \frac{1}{5}\begin{pmatrix} 13 & 14 \\ 14 & 33 \end{pmatrix}$$

$8, \underline{1}$

$\ker((A - I))$ $\qquad (A-I)\vec{v} = 0$

$\Updownarrow$

$\underbrace{\ker(5A - 5I)}$ $\qquad 5(A-I)\vec{v} = 0$

$$\ker\left(\left(\begin{pmatrix} 12 & 14 \\ 14 & 33 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\right)v\right)$$

$$= \ker\begin{pmatrix} 7 & 14 \\ 14 & 28 \end{pmatrix}$$

$R_2 = 2R_1$

$\cancel{5A - I}$

$5A$ eigenvalues $40, 5$

40 - eigenspace of $5A$ = 8 - eigenspace of A

5 " " $5A$ = 1 - " " " A

$$A = \frac{1}{5}\begin{pmatrix} 12 & 14 \\ 14 & 33 \end{pmatrix}$$

$$\ker\left(\boxed{\frac{1}{5}\begin{pmatrix} 12 & 14 \\ 14 & 33 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}\right)$$

$$\| \qquad \frac{12}{5} - 1$$

$$\ker\left(\begin{pmatrix} 12 & 14 \\ 14 & 33 \end{pmatrix} - 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$\ker\left(\frac{1}{5}\begin{pmatrix} 12 & 14 \\ 14 & 33 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$= \ker\begin{pmatrix} 7/5 & 14/5 \\ 14/5 & 28/5 \end{pmatrix} \qquad R_2 = 2R_1$$

$$= \ker\begin{pmatrix} 1 & 2 \\ 14/5 & 28/5 \end{pmatrix} \qquad R_1/\left(\frac{7}{5}\right)$$

$$= \ker\begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} \qquad R_2 - \frac{14}{5}R_1$$

$$x + 2y = 0$$

$$= \operatorname{Span}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\ker(A - I)$$

$$\parallel$$

$$\ker(S(A-I))$$

$(A-I)\vec{v}=\vec{0} \iff S(A-I)\vec{v}=\vec{0}$

$(\Rightarrow)$

$(A-I)\vec{v}=\vec{0} \Rightarrow S(A-I)\vec{v}=S\cdot\vec{0}$

$\Rightarrow S(A-I)\vec{v}=\vec{0}$

$\Rightarrow \vec{v}$ in $\ker(S(A-I))$

$(\Leftarrow)$ $\dfrac{S(A-I)\vec{v}=\vec{0}}{S}$

$(A-I)\vec{v}=\vec{0}$

$\therefore \vec{v}\in\ker(A-I)$