$$\begin{pmatrix} \lambda & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda \end{pmatrix} \approx \text{diagonal}$$

"Jordan canonical form"

$\boxed{\text{SVD}}$

orthonormal diagonalizable

$\Updownarrow$

symmetric

$\approx \frac{1}{2}$ dimension all matrices

A $n \times m$

$A^T A$ $m \times m$

$(A^T A)^T = A^T A^{TT} = A^T A$

symmetric

Spectral thm

ONB eigenvectors of $A^T A$

$\vec{v}_1, \dots, \vec{v}_m$

$\vec{v} \cdot \vec{w} = \vec{v}^T \vec{w}$

$$\boxed{A^T A \vec{v}_i = \lambda_i \vec{v}_i}$$

$\vec{v}_i \cdot (A^T A \vec{v}_i)$

$= \underbrace{\vec{v}_i^T}_{1 \times m} \underbrace{(A^T A \vec{v}_i)}_{m \times 1}$ $(\vec{v}_i^T A^T)(A \vec{v}_i)$

$(\cdot \cdot \dots \cdot) \begin{pmatrix} \cdot \\ \cdot \\ \vdots \\ \cdot \end{pmatrix}$

$$\underbrace{\vec{v}_i}_{m\times 1} \cdot \underbrace{\left(A^T A \vec{v}_i\right)}_{m\times 1} = \underbrace{\vec{v}_i^{\,T}}_{1\times m} \underbrace{\left(A^T A \vec{v}_i\right)}_{m\times 1}$$

$$= \left(\vec{v}_i^{\,T} A^T\right)\left(A \vec{v}_i\right)$$

$$= \left(A\vec{v}_i\right)^T A\vec{v}_i$$

$$\underbrace{\vec{w}^{\,T}}_{1\times m} \cdot \underbrace{\vec{w}}_{m\times 1} = \vec{w} \cdot \vec{w}$$

$$\vec{w}^{\,T} \vec{w}^{\,TT} = \left(\vec{w}^{\,T} \vec{w}\right)^T$$

$$\begin{pmatrix} w_1 & \dots & w_m \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_m \end{pmatrix}$$

$$= w_1^2 + w_2^2 + \dots + w_m^2$$

$$= \|\vec{w}\|^2 \geq 0$$

$$= \|A\vec{v}_i\|^2 \geq 0$$

$$= \vec{v}_i \cdot \left(\lambda_i \vec{v}_i\right)$$

$$= \lambda_i \left(\vec{v}_i \cdot \vec{v}_i\right)$$

$$= \lambda_i \|\vec{v}_i\|^2$$

$$\lambda_i \|\vec{v}_i\|^2 = \|Av_i\|^2$$

$$A^TA\,\vec{v}_i = \lambda_i \vec{v}_i \qquad \|\vec{v}_i\|^2 = 1$$

$$\vec{v}_i \neq \vec{0}$$

$$\boxed{\lambda_i = \frac{\|Av_i\|^2}{\|v_i\|^2} \geq 0} \qquad \lambda_i = \underbrace{\|Av_i\|^2}$$

eigenvalues of A^TA are nonnegative

$$\begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$$

A^TA "positive <u>semi</u>definite

$$\lambda_i = \|A v_i\|^2$$

$$(A\vec{v}_i)\cdot(A\vec{v}_j)$$

$$(A\vec{v})\cdot(\vec{w}) = \vec{v}^T A^T \vec{w}$$

$$= \vec{v}^T (A^T \vec{v})$$

$$= \vec{v}\cdot(A^T\vec{w})$$

$$(\underline{A\vec{v}})\cdot\vec{w} = \vec{v}\cdot(A^T\vec{w})$$

$$(\underline{A\vec{v}_i})\cdot(Av_j) = \vec{V}_i \cdot (A^T A \vec{v}_j)$$

$$= \vec{V}_i \cdot (\lambda_j \vec{V}_j)$$

$$= \lambda_j (\vec{v}_i \cdot \vec{v}_j)$$

$$= \begin{cases} \lambda_j & i = j \\ 0 & i \neq j \end{cases}$$

$i \neq j$, $(A\vec{v}_i) \cdot (A\vec{v}_j) = 0$

Moral: A takes ONB of eigenvectors of A^TA to some orthogonal set.

$\lambda_i = \|A\vec{v}_i\|^2$

$\sigma_i = \sqrt{\lambda_i} = \|A\vec{v}_i\|$

$\vec{u}_i = \dfrac{Av_i}{\sigma_i}$ if $\sigma_i \neq 0$
if $\vec{Av_i} \neq 0$

$\|\vec{u}_i\| = 1$

$\vec{u}_i \cdot \vec{u}_j = \begin{cases} 1 & i = j \\ 0 & \text{o/w} \end{cases}$

$\vec{u}_1, \ldots, \vec{u}_r$ so far

$\vec{u}_1, \ldots, \vec{u}_n$ ONB

A $n \times m$ matrix

$v_i \in \mathbb{R}^m \xrightarrow{A} \mathbb{R}^n \ni u_i$

$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \underline{\begin{pmatrix} 0 \\ 0 \end{pmatrix}} \right\}$ orthogonal

$\vec{v}_1, \ldots, \vec{v}_m$ in $\mathbb{R}^m$ ONB eigenvectors $A^T A$

$A^T A \vec{v}_i = \lambda_i \vec{v}_i$

$\lambda_i = \|A v_i\|^2$

$$(A\vec{v}_i) \cdot (A\vec{v}_j) = \begin{cases} \lambda_i & i = j \\ 0 & i \neq j \end{cases}$$

$\{A v_1, \ldots, A v_m\}$ orthogonal set

$\lambda_1 \geq \lambda_2 \geq \lambda_3 \ldots \geq \lambda_m$

$\lambda_1 = 4, \lambda_2 = 5$

swap v_1, v_2

$v_1' = v_2, \quad v_2' = v_1$

$$\lambda_1 \geq \lambda_2 \ldots \geq \underbrace{\lambda_r}_{\text{last nonzero}} \overbrace{\geq \underset{\lambda_{r+1}}{\underset{\parallel}{0}} \geq \underset{\lambda_{r+2}}{\underset{\parallel}{0}} \ldots \geq \underset{\lambda_m}{\underset{\parallel}{0}}}$$

$$\underset{\substack{\shortparallel \\ v_1}}{\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}}, \quad \underset{v_2}{\begin{pmatrix}-1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}}$$

$$A^T A v_1 = v_1$$

$$A^T A v_2 = 2v_2$$

$$v_1 \longleftrightarrow v_2$$

$$v_1' = v_2$$

$$v_2' = v_1$$

$$A^T A v_1' = 2v_1'$$

$$A^T A\ v_2' = v_2'$$

$$\underset{\substack{\shortparallel \\ \lambda_1}}{2} \geq \underset{\substack{\shortparallel \\ \lambda_2}}{1}$$

If $1 \le i \le r$

$$\boxed{\vec{u_i} \underset{def}{=} \frac{A\vec{v_i}}{\|A\vec{v_i}\|}}$$

$$\|A\vec{v_i}\| = \sqrt{\lambda_i} \neq 0$$

$\vec{u_1}, \dots, \vec{u_r}$ orthonormal set

extend this $\underbrace{\vec{u_1}, \dots, \vec{u_r}}_{\text{above procedure}}, \underbrace{\vec{u_{r+1}}, \dots, \vec{u_n}}_{\text{choose}}$

$$A\vec{v_i} = \underbrace{\sqrt{\lambda_i}}_{\sigma_i = \sqrt{\lambda_i}} \vec{u_i} \quad \text{if } 1 \le i \le r$$

$$A\vec{v_i} = \sigma_i \vec{u_i} \qquad 1 \le i \le r$$

$$A\vec{v_i} = \vec{0} \qquad i = r+1, r+2, \dots, n$$

$$A \underbrace{\begin{pmatrix} | & & | & | & & | \\ v_1 & \cdots & v_r & v_{r+1} & \cdots & v_m \\ | & & | & | & & | \end{pmatrix}}_{V}$$

$$= \begin{pmatrix} | & & | \\ Av_1 & \cdots & Av_m \\ | & & | \end{pmatrix}$$

$$= \begin{pmatrix} | & & | & | & | & & | \\ \sigma_1 \vec{u}_1 & \cdots & \sigma_r \vec{u}_r & \vec{0} & \vec{0} & \cdots & \vec{0} \\ | & & | & | & | & & | \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} | & & | \\ u_1 & \cdots & u_n \\ | & & | \end{pmatrix}}_{U} \underbrace{\begin{pmatrix} \sigma_1 & & 0 \\ & \ddots & \\ & & \sigma_r \\ 0 & 0 & 0 \\ 0 & 0 & \cdot \\ & \ddots & 0 \end{pmatrix}}_{\Sigma}$$

$$\underset{n\,m}{A}\,\underset{m\,m}{V} = \underset{n\,n}{U}\,\underset{n\,m}{\Sigma} \qquad \begin{pmatrix} \sigma_1 & & & \\ & \ddots & & \\ & & \sigma_r & 0 \end{pmatrix}$$

$$A = U \Sigma V^T$$

A $n \times m$

V $m \times m$ orthog

U $n \times n$ orthog

Σ "diagonal" $n \times m$
↳ entries ≥ 0

$$A = \underline{U}\,\underline{\Sigma}\,\underline{V^T}$$

$$\begin{tikzcd} \mathbb{R}^m \arrow[r] \arrow[d, "V^T\ \text{rigid}"'] & \mathbb{R}^n \\ \cdot \arrow[r, "\text{scale along axes}", "\Sigma"'] & \cdot \arrow[u, "\text{rigid}"'] \end{tikzcd}$$

3. a) A $n \times n$

$$A = \underbrace{U}_{\text{orthog}}\,\underbrace{\Sigma}_{\text{diagonal}}\,\underbrace{V^T}_{\text{orthog}}$$

$$(\Sigma V^T)^T = V\Sigma^T$$

$$\neq \Sigma V^T$$

$$(V\Sigma V^T)^T = V\Sigma V^T$$

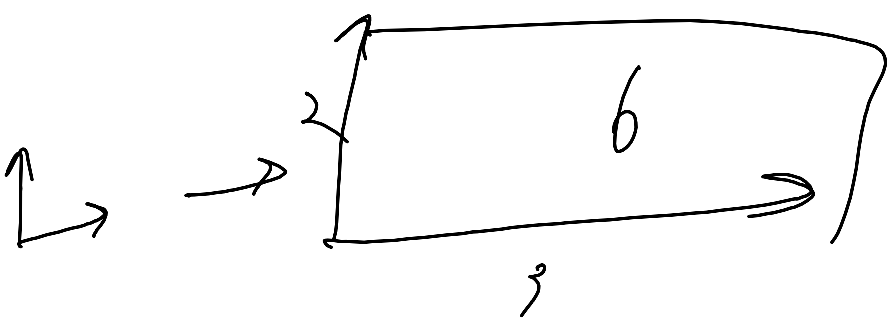
$V\Sigma V^T$ sym pos semidef

$$A = U\Sigma V^T$$

$$V\Sigma V^T$$

$$= U \underbrace{V^T V}_{I} \Sigma V^T$$

$$= (UV^T)(V\Sigma V^T)$$



$$A = \begin{pmatrix} 0 & 1 \\ -5 & 4 \end{pmatrix}$$

$$a.m(\lambda) = 1 \Downarrow g.m(\lambda) = 1$$

e.v. $2 \pm i$

find $\vec{v} + i\vec{w}$

$$A(\underbrace{\vec{v} + i\vec{w}}_{\mathbb{C}^2}) = (2+i)(\vec{v} + i\vec{w})$$

$\vec{v} - i\vec{w}$ e.v. $2 - i$

$\vec{v}, \vec{w}$ $\mathbb{R}^2$

$$p_A(\lambda) = \lambda^2 - 4\lambda + 5$$
$$= (\lambda - \underbrace{(2+i)}_{a.m = 1})(\lambda - \underbrace{(2-i)}_{a.m = 1})$$

$$1 \le \underset{\dim(A - \lambda I)}{\overset{\shortparallel}{g.m(\lambda)}} \le a.m(\lambda) = 1$$

$2+i$ - e.v.

$\ker\left(\underline{A-(2+i)I}\right)$

$\ker\left(\begin{pmatrix} 0 & 1 \\ -5 & 4 \end{pmatrix} - \begin{pmatrix} 2+i & 0 \\ 0 & 2+i \end{pmatrix}\right)$

$= \begin{pmatrix} -2-i & 1 \\ -5 & 4-(2+i) \end{pmatrix}$

$(-2-i)x_1 + x_2 = 0$

$= \begin{pmatrix} -2-i & 1 \\ -5 & 2-i \end{pmatrix}$

$\vec{v}$ eigenvect of e.v. 0

$\Updownarrow$

$A\vec{v} = \vec{0}$

$\vec{v} \in \ker(A)$

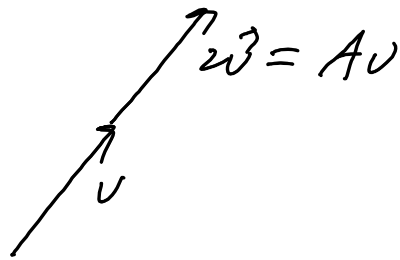
$\det(\underline{A-\lambda I}) = 0 \iff$ root of char poly

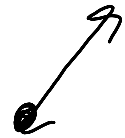
$\Updownarrow$

$\boxed{\ker(\underline{A-\lambda I})} \neq \{\vec{0}\}$

$\Updownarrow$

there is some $\vec{v}$ nonzero s.t. $A\vec{v} = \lambda\vec{v}$



$A - 2v$ 

$$\text{null}\begin{pmatrix} \dfrac{-2-i}{-5} & \dfrac{1}{2-i} \end{pmatrix} = 1$$

$$\begin{pmatrix} \geq 1 & \det = 0 \\ \leq 1 & g.m\,(2+i) \doteq 1 \end{pmatrix}$$

$\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$, $(2-\lambda)^2$ $\quad$ $a.m\,(2) = 2$

A'' $\quad$ $g.m\,(2) = 1$

$A - 2I = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} \underline{-2-i} & 1 \\ -5 & 2-i \end{pmatrix}$$

$$\Big\downarrow \; R_1 \mapsto \frac{R_1}{-2-i}$$

$$\begin{pmatrix} 1 & -\frac{1}{2+i} \\ -5 & 2-i \end{pmatrix}$$

$$\Big\downarrow \; R_2 + 5R_1$$

$$\begin{pmatrix} 1 & -\frac{1}{2+i} \\ 0 & \boxed{(2-i)+5\left(\frac{-1}{2+i}\right)} \end{pmatrix}$$

$$2 - i - \frac{5}{2+i}$$

$$2-i-\frac{5i}{2+i}$$

$$=2-i-\frac{5i(2-i)}{(2+i)(2-i)}$$

$$=2-i-\frac{5i(2-i)}{5}$$

$$=2-i-\frac{5}{5}i(2-i)$$

$$=(2-i)-(2-i)$$

$$=0$$

$$\begin{pmatrix} 1 & \frac{-1}{2+i} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x_1 - \frac{x_2}{2+i} = 0$$

$$0 = 0$$

$$t = x_2$$

$$x_1 - \frac{t}{2+i} = 0$$

$$x_1 = \frac{t}{2+i}$$

some scalar

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} t/(2+i) \\ t \end{pmatrix} = \boxed{t \begin{pmatrix} \frac{1}{2+i} \\ 1 \end{pmatrix}}$$

basis for $\ker(A-(2+i)I)$

$$\operatorname{Span}\begin{pmatrix} \frac{1}{2+i} \\ 1 \end{pmatrix} = \operatorname{Ker}(A-(2+i)I)$$

$$= \operatorname{Span}\begin{pmatrix} 1 \\ 2+i \end{pmatrix}$$

$$\boxed{\begin{pmatrix} \lambda & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda \end{pmatrix} \quad \begin{array}{l} a.m = 4 \\ g.m = 1 \end{array}}$$

Jordan block

$$S^{-1}AS = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

a, b real #s

$$\underline{A \begin{pmatrix} 1 \\ 2+i \end{pmatrix}} = (2+i) \begin{pmatrix} 1 \\ 2+i \end{pmatrix}$$

$$\underline{A \begin{pmatrix} 1 \\ 2-i \end{pmatrix}} = (2-i) \begin{pmatrix} 1 \\ 2-i \end{pmatrix}$$

$$\vec{z} = \begin{pmatrix} 1 \\ 2+i \end{pmatrix}, \quad \lambda = 2+i$$

$$Az = \lambda z$$

$$A\bar{z} = \bar{\lambda}\bar{z}$$

$$\bar{z} = \begin{pmatrix} 1 \\ 2-i \end{pmatrix}$$

$$\bar{\lambda} = 2-i$$

$$Az = \lambda z$$

$$\overline{Az} = \overline{\lambda z}$$

$$\overline{Az} = \bar{A}\bar{z} \qquad \overline{\lambda z} = \underline{\underline{\bar{\lambda}\bar{z}}}$$

$$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$$\bar{A}\bar{z} = \overline{\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}}$$

$$= \overline{\begin{pmatrix} az_1 + bz_2 \\ cz_1 + dz_2 \end{pmatrix}}$$

$$= \begin{pmatrix} \bar{a}\bar{z}_1 + \bar{b}\bar{z}_2 \\ \bar{c}\bar{z}_1 + \bar{d}\bar{z}_2 \end{pmatrix}$$

$$= \bar{A}\bar{z}$$

$$\overline{\lambda z} = \overline{\begin{pmatrix} \lambda z_1 \\ \lambda z_2 \end{pmatrix}} = \begin{pmatrix} \overline{\lambda z_1} \\ \overline{\lambda z_2} \end{pmatrix} = \begin{pmatrix} \bar{\lambda}\bar{z}_1 \\ \bar{\lambda}\bar{z}_2 \end{pmatrix}$$

$$\overline{ab} = \bar{a}\,\bar{b}$$

$$\overline{5+0i} = 5-0i = 5$$

$$\cdot\ \overline{w} = w$$

$$\Updownarrow$$

w is a real number

$$\overline{A} = A$$

$$A\overline{z} = \overline{\lambda}\,\overline{z}$$

$\underline{\underline{z}}, \underline{\underline{\overline{z}}}$ $\quad\quad$ $\lambda \neq \overline{\lambda}$

$\lambda, \overline{\lambda}$

$\underline{c_1} z + \underline{c_2} \overline{z} = 0 \quad \Rightarrow c_1, c_2 = 0$

c_i in $\mathbb{R}$

$\underline{c_i}$ in $\underline{\mathbb{C}}$

A similar to $\underline{\begin{pmatrix} 2+i & 0 \\ 0 & 2-i \end{pmatrix}}$

$$P = \begin{pmatrix} z & \bar{z} \\ 1 & 1 \end{pmatrix}$$

$$\underbrace{P^{-1}}_{\mathbb{C}}\underbrace{A}_{\mathbb{R}}\underbrace{P}_{\mathbb{C}} = \underbrace{\begin{pmatrix} 2+i & 0 \\ 0 & 2-i \end{pmatrix}}_{\mathbb{C}}$$

$\begin{pmatrix} 2+i & 0 \\ 0 & 2-i \end{pmatrix}$ similar to $\underline{\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}}$

$$R = \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2+i & 0 \\ 0 & 2-i \end{pmatrix} = R^{-1}\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} R$$

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$\det \begin{pmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{pmatrix}$$

$$\underbrace{(2-\lambda)^2} \; \underbrace{+1} = 0$$

$$= 4 - 4\lambda + \lambda^2 + 1$$

$$= \lambda^2 - 4\lambda + 5$$

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} i \\ 1 \end{pmatrix} = \begin{pmatrix} 2i - 1 \\ i + 2 \end{pmatrix}$$

$$= \lambda \begin{pmatrix} i \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda i \\ \lambda \end{pmatrix}$$

$$\lambda = i + 2$$

$$B = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

ch.p's $= \lambda^2 - 4\lambda + 5$

e.v. $2 \pm i$

$$B - (2+i)I = \begin{pmatrix} 2-(2+i) & -1 \\ 1 & 2-(2+i) \end{pmatrix}$$

$$= \begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix}$$

$$R_2 = iR_1$$

$$-i x_1 - x_2 = 0$$

$$x_1 = i$$

$$-i^2 - x_2 = 0$$

$$1 - x_2 = 0, \quad x_2 = 1$$

$$\begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix} \begin{pmatrix} i \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$B\begin{pmatrix} -i \\ 1 \end{pmatrix} = (2-i)\begin{pmatrix} -i \\ 1 \end{pmatrix}$$

$$B\begin{pmatrix} i \\ 1 \end{pmatrix} = (2+i)\begin{pmatrix} i \\ 1 \end{pmatrix}$$

A similar to $\underline{\begin{pmatrix} 2+i & 0 \\ 0 & 2-i \end{pmatrix}}$

$$P = \begin{pmatrix} \frac{1}{z} & \frac{1}{\bar{z}} \\ 1 & 1 \end{pmatrix}$$

$$\underbrace{P^{-1}}_{\mathbb{C}} \underbrace{A}_{\mathbb{R}} \underbrace{P}_{\mathbb{C}} = \underbrace{\begin{pmatrix} 2+i & 0 \\ 0 & 2-i \end{pmatrix}}_{\mathbb{C}}$$

$\begin{pmatrix} 2+i & 0 \\ 0 & 2-i \end{pmatrix}$ similar to $\underbrace{\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}}_{\mathbb{R}}$

$$R = \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2+i & 0 \\ 0 & 2-i \end{pmatrix} = R^{-1} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} R$$

$$P^{-1}AP = \begin{pmatrix} 2+i & 0 \\ 0 & 2-i \end{pmatrix} = R^{-1}BR$$

$$S^{-1}AS = B = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$P^{-1}AP = R^{-1}BR$$

$$RP^{-1}AP = BR$$

$$RP^{-1}APR^{-1} = B$$

$$(RP^{-1})A(PR^{-1}) = B$$

$$(PR^{-1})^{-1} = RP^{-1}$$

$$\underbrace{(PR^{-1})^{-1}}_{S^{-1}} A \underbrace{(PR^{-1})}_{S} = B \qquad \text{let } S = PR^{-1}$$

$$S^{-1}AS = B$$

$$S = PR^{-1}$$

$$= \begin{pmatrix} \overset{1}{z} & \overset{1}{\bar{z}} \\ 1 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix}^{-1}}_{}$$

$$\det = 2i$$

$$\frac{1}{2i} \begin{pmatrix} 1 & i \\ -1 & i \end{pmatrix}$$

$$\frac{1}{2i} \begin{pmatrix} \overset{1}{z} & \overset{1}{\bar{z}} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & i \\ -1 & i \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \ \vec{w} = \begin{pmatrix} i \\ i \end{pmatrix}$$

$$z = \vec{v} + i\vec{w} \qquad \vec{v}, \vec{w} \ \underline{\text{real}}$$

$$\bar{z} = \vec{v} - i\vec{w}$$

$$\frac{1}{2i} \begin{pmatrix} \vec{v} + i\vec{w} & \vec{v} - i\vec{w} \end{pmatrix} \begin{pmatrix} 1 & i \\ -1 & i \end{pmatrix}$$

$$\frac{1}{2i}\begin{pmatrix} z & \bar{z} \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & i \\ -1 & i \end{pmatrix}$$

$$\frac{1}{2i}\begin{pmatrix} z & \bar{z} \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \frac{1}{2i}\left(1 \cdot z - 1 \cdot \bar{z}\right)$$

$$\frac{z - \bar{z}}{2i}$$

$$\frac{(v + iw) - (v - iw)}{2i}$$

$$\frac{2iw}{2i} = w$$

$$\frac{1}{2i}\begin{pmatrix} z & \bar{z} \\ 1 & 1 \end{pmatrix}\begin{pmatrix} i \\ i \end{pmatrix}$$

$$\frac{iz + i\bar{z}}{2i}$$

$$\frac{z+\bar{z}}{2}$$

$$= \frac{(v+iw) + (v-iw)}{2}$$

$$= \frac{2v}{2}$$

$$= v$$

$$PR^{-1} = \begin{pmatrix} w & v \\ 1 & 1 \end{pmatrix} \quad \text{real}$$

$$\int$$

A 2×2 real

- e.v. $a \pm bi$ char poly stuff
- e.vect $v \pm iw$ for stuff

$$S = \begin{pmatrix} w & v \end{pmatrix}$$

$$S^{-1} A S = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

---

e.v. A $2 \pm i$

$a = 2$ $b = 1$

$v = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $w = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 2+i \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}_{v} + i \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{w}$

$S = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$ $2+i \rightsquigarrow v+iw$

$2-i \rightsquigarrow v-iw$

$$\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 0 & 1 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$