Char polys

A $n \times n$

$p(\lambda) = \det(A - \lambda I)$.

$p(0) = \det(A)$

$p = a_0 + a_1 \lambda + \cdots + a_{n-1} \lambda^{n-1} + (-1)^n \lambda^n$

$p(0) = a_0$

$a_0 = \det(A)$.

$$\begin{pmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & & \vdots \\ a_{31} & & \ddots & \vdots \\ \vdots & & & a_{nn} - \lambda \\ a_{n1} & \cdots & & \end{pmatrix}$$

$$p(\lambda) = \det(A - \lambda I)$$

$$p(\lambda) = \prod_{i=1}^{n} (r_i - \lambda)$$

$r_1, \dots, r_n$ roots of p

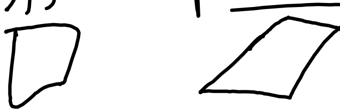
$$p = \lambda^2$$

$$(r_1 - \lambda)(r_2 - \lambda) \cdots (r_n - \lambda)$$

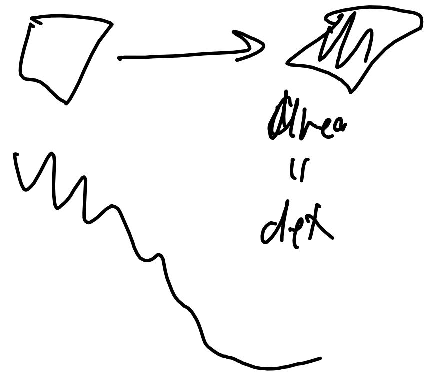
$$p(0) = r_1 r_2 \cdots r_n$$

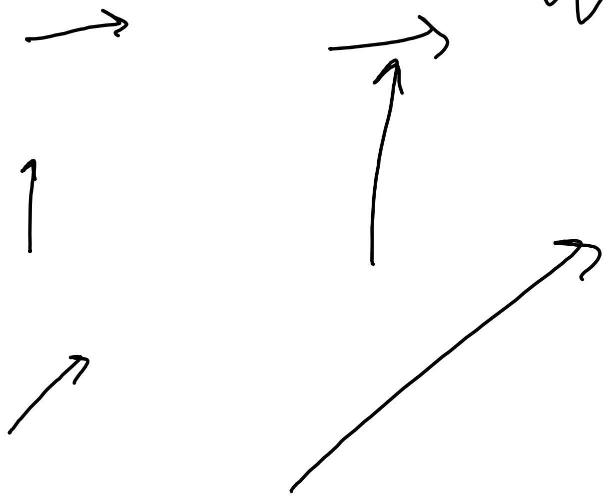
r_i eigenvalues of A.

$\det(A)$ = product of eigenvalues, including multiplicity



$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$





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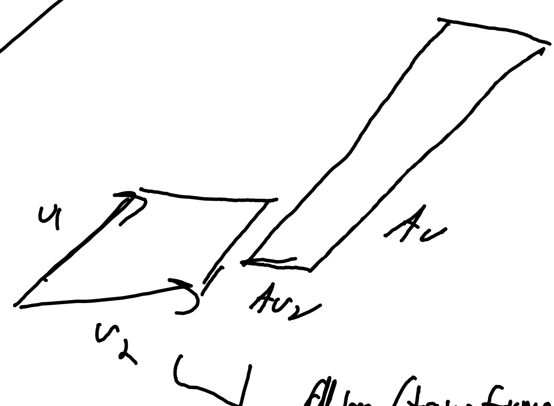

$v_1, \ldots, v_n$ basis

$Av_i = \lambda_i v_i$

$$\det(A) = \frac{\text{area(transformed parallelogram)}}{\text{area(parallelogram)}}$$

$$\det(S^{-1}AS) = \det(\cancel{S^{-1}})\det(A)\cancel{\det(S)}$$
$$= \det(A)$$

$$p(\lambda) = \det(\underline{A - \lambda I}) \quad \left(\begin{array}{l}\text{aside} \\ \text{def} \quad \det(\lambda I - A)\end{array}\right)$$

$$p = \prod (r_i - \lambda) = (r_1 - \lambda)(r_2 - \lambda)\cdots(r_n - \lambda)$$

r_i eigenvalues w/ alg mult

coeff of λ^{n-1}

$$p(0) = \det(A)$$
$$\| \quad =$$
$$\prod r_i$$

$$(\underline{r_1} - \underline{\lambda})(\underline{r_2 - \lambda})(\underline{r_3} - \underline{\lambda})$$

λ^2 coefficient: $r_1(-\lambda)(-\lambda) + (-\lambda)r_2(-\lambda) + (-\lambda)(-\lambda)r_3$

$$= (r_1 + r_2 + r_3)\lambda^2$$

$$P = \prod (r_i - \lambda)$$

λ^{n-1} coefficient

$$\underline{r_1} - \lambda)(r_2 - \lambda) \cdots (r_n - \lambda)$$

$$r_1 (-\lambda)^{n-1} + (-\lambda) r_2 (-\lambda)^{n-2} + \cdots + (-\lambda)^{n-1} r_n$$

$$= \underbrace{\underbrace{\left(\sum_{i=1}^{n} r_i\right)} \underline{(-1)^{n-1}}} \lambda^{n-1}$$

$tr(A)$ = sum of eigenvalues, counted w/ multi

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \quad tr(A) = 1+2+3$$

$$= 1+2+3$$

$$tr(S^{-1}AS) \underline{\overset{?}{=}} tr(A)$$

$$p(\lambda) = (-\lambda)^n + (-1)^{n-1} \underline{tr(A)}\, \lambda^{n-1} + \dots + \underline{det(A)}$$

$$\prod_{i=1}^{n} (r_i - \lambda)$$

λ coefficient

$$(r_1 - \lambda)(r_2 - \lambda)\dots(r_n - \lambda)$$

$$(-\lambda)(r_2)(r_3)\dots(r_n)$$

$$+\, r_1(-\lambda)(r_3)\dots(r_n)$$

"multilinear algebra"

$$tr\left(\Lambda^{n-1}\mathbb{R}^n \xrightarrow{\Lambda^{n-1}A} \Lambda^{n-1}\mathbb{R}^n\right)$$

$$\vdots \qquad \ddots$$

$$+\, r_1 r_2 \dots r_{n-1}(-\lambda)$$

$$(-1)\underbrace{\left(\underbrace{r_2 r_3 \dots r_n}_{\text{omit } r_1} + \underbrace{r_1 r_3 \dots r_n}_{\text{omit } r_2} + \underbrace{r_1 r_2 r_4 \dots r_n}_{\text{omit } r_3} + \dots + \underbrace{r_1 \dots r_{n-1}}_{\text{omit } r_n}\right)}$$

λ^2

$\prod (r_i - \lambda)$

λ^2 coeff sum ~~of~~ all products of the r_i where you omit two indices

$\vdots$

λ^k omit k many indices

"elementary symmetric polynomials"

"Vieta formulas"

$\boxed{\det(A - \lambda I)}$

1. 115 A, AH, B

"Jordan Canonical form"

$n \times n$ matrix

$$J_n(k) = \begin{pmatrix} k & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & k \end{pmatrix} \qquad \vec{e}_1$$

$$\det(J_n(k) - \lambda I) = \det \begin{pmatrix} k-\lambda & 1 & & \\ & k-\lambda & \ddots & \\ & & \ddots & 1 \\ & & & k-\lambda \end{pmatrix}$$

$$= (k-\lambda)^{\underline{n}}$$

only root (eigenvalues of $J_n(k)$) is k, alg mult n

mult $2 \rightarrow (2-\lambda)^2 (3-\lambda) \leftarrow 1$ mult

Geometric multiplicity

$p(\lambda) = \det(A - \lambda I)$

$p(\lambda) = 0 \iff \det(A - \lambda I) = 0$

$\iff A - \lambda I$ not invertible

$\iff \ker(A - \lambda I) \neq \{\vec{0}\}$

$\iff$ there is some nonzero $\vec{v}$ in $\ker(A - \lambda I)$

$\iff$ there is nonzero $\vec{v}$ s.t. $\underset{\displaystyle Av - \lambda v}{\underset{\|}{(A - \lambda I)\vec{v}}} = 0$

$\iff Av = \lambda v$

λ-Eigenspace

$\|$

$\ker(A - \lambda I)$ = solution for $A\vec{v} = \lambda\vec{v}$

Def. geometric multiplicity of the eigenvalue λ of A is $\dim \ker(A - \lambda I) = \text{null}(A - \lambda I)$

$\boxed{\text{geometric multiplicity} \leq \text{algebraic multiplicity}}$

geom mult of e.v. k in $J_n(k)$?

$$\ker\left(J_n(k) - kI\right)$$

$$\begin{pmatrix} k & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & k \end{pmatrix} - \begin{pmatrix} k & & \\ & \ddots & \\ & & k \end{pmatrix}$$

$$B = \left[\begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & 0 \end{pmatrix}\right] \qquad \vec{e}_1$$

$rk(B) = n-1$

rank-nullity.

$null(B) = 1$

$$\underline{rk(B)} + \underline{null(B)} = n$$

$\parallel$

geom mult of e.v. k in $J_n(k)$

geom mult 1 <<<<< alg mult n

goal: diagonalize as much as possible

$\Updownarrow$

similar $\begin{pmatrix} * & & 0 \\ & \ddots & \\ 0 & & * \end{pmatrix}$

$\Updownarrow$

basis of eigenvectors

$\Updownarrow$

$\sum$ geom mult $= n$

always know $\sum$ alg mult $= n$ ← deg char poly

$(1-\lambda)^2(3-\lambda)$

geom $\leq$ alg

each alg mult$_\lambda$ = geom mult λ

$$\begin{pmatrix} k & 1 & & & & \\ & k & & & & \\ & & k & & & \\ & & & k & & \\ & & & & \ddots & \\ & & & & & k \end{pmatrix}$$

geom mult $n-1$

alg mult n