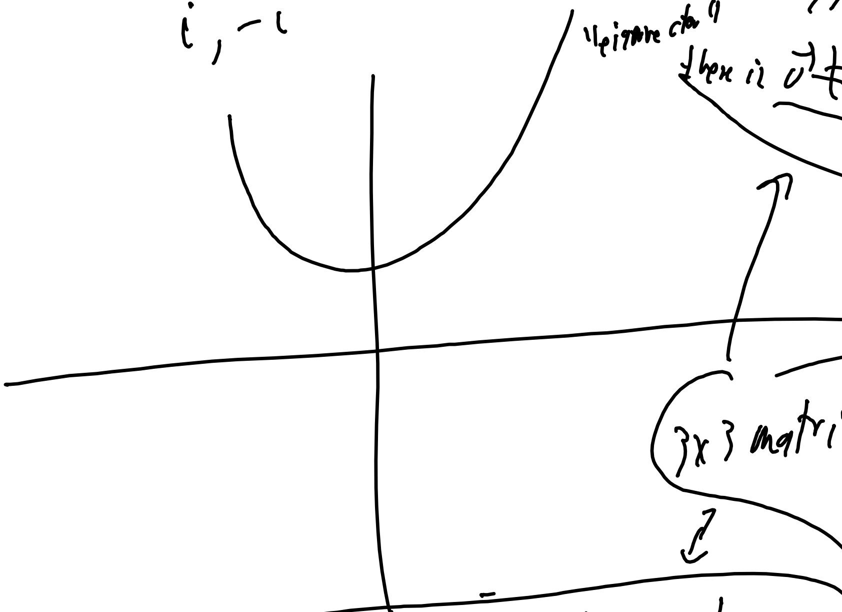
1. $x^2+1=0$

$i, -i$



$\lambda \in \mathbb{R}$ is an eigenvalue of $A_{n\times n}$ if there is $\vec{v} \neq \vec{0}$ s.t. $A\vec{v} = \lambda \vec{v}$

$\vec{v}$ "eigenvector"

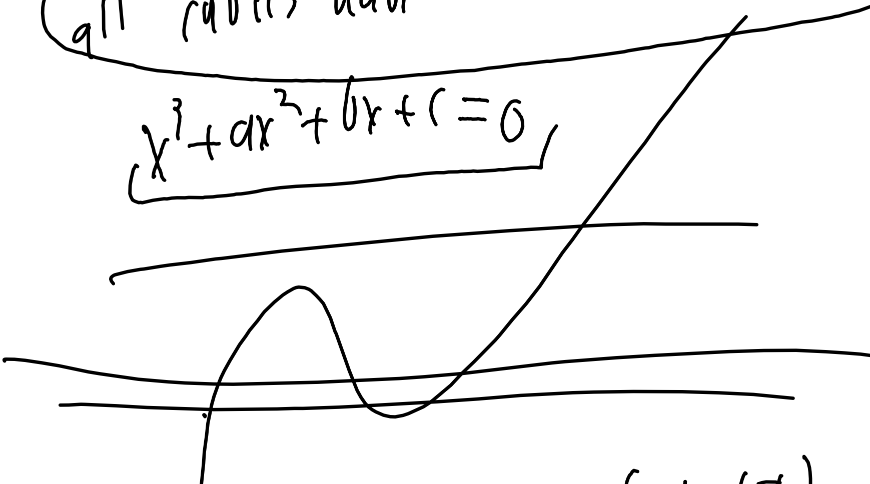
3x3 matrices have a real eigenvalue

mystery!?!?

$\Updownarrow$

all cubics have a real root

$x^3+ax^2+bx+c=0$



$p(t)=\det(A-tI)$ has zero

$p(\lambda)=0$, $A-\lambda I$ has a kernel

2. V a subspace of $\mathbb{R}^n$

Definition A $n\times n$ matrix

V is A-invariant

if $\vec{v}$ in $V \Rightarrow A\vec{v}$ in V

$n \geq 1$

$\operatorname{span}\left\{\begin{pmatrix}1\\2\\ \vdots\\ n\end{pmatrix}\right\}$

A fixed matrix

$\operatorname{im}(A) = \{A\vec{w} \mid \vec{w} \text{ in } \mathbb{R}^n\}$ inv. subspace

$\vec{v}$ in $\operatorname{im}(A)$. $\vec{v} = A\vec{w}$ some $\vec{w}$ $\mathbb{R}^n$

$\Rightarrow A\vec{v}$ in $\operatorname{im}(A)$

$A = \begin{pmatrix}\vec{c}_1 & \dots & \vec{c}_n\end{pmatrix}$

$A\vec{v} = \sum v_i \vec{c}_i$

$A\vec{v} \in$ span columns $= \operatorname{im}(A)$

$\ker(A)$?

V subspace: $\vec{0}$ in V

$\vec{v}, \vec{w}$ in $V \Rightarrow \vec{v} + \vec{w}$ in V

$\lambda \in \mathbb{R}$, $\vec{v}$ in $V \Rightarrow \lambda \vec{v}$ in V

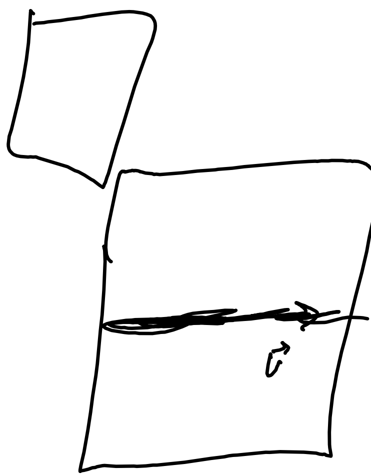
I_n, $\operatorname{im}(I_n) = \mathbb{R}^n$

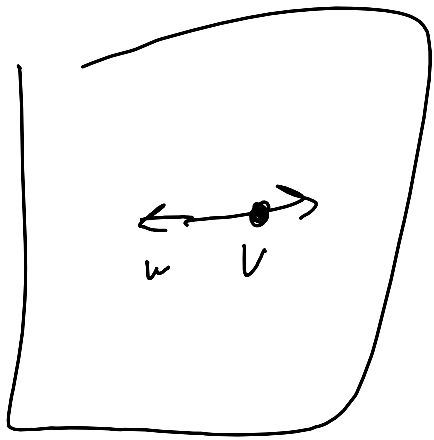
$\mathbb{R}^n$ is an I_n-invariant subspace

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ 90° ccwise rotation

What are the A-invariant subspaces?

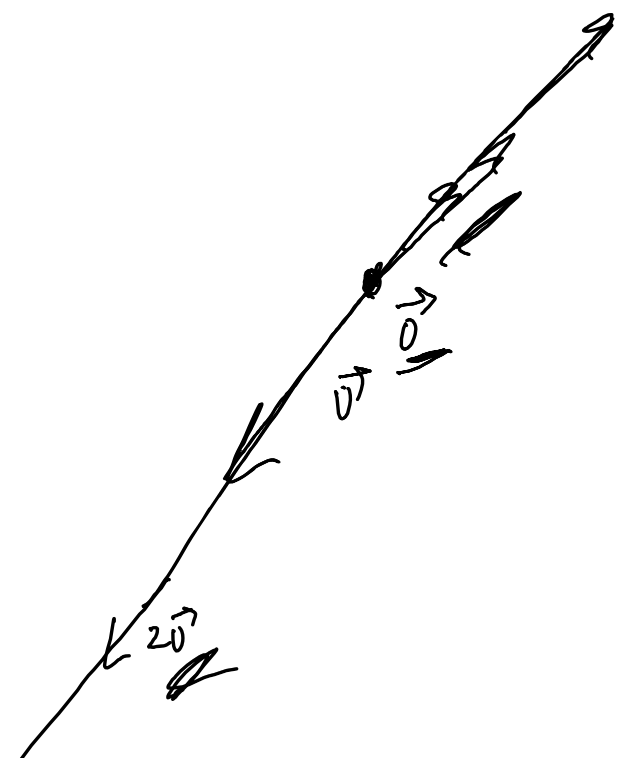
subspaces of $\mathbb{R}^2$





e.g. line through the origin is a subspace of $\mathbb{R}^2$

dim 1 subspace of $\mathbb{R}^2$

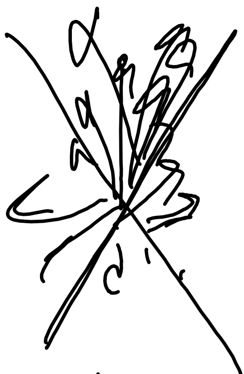


Stratify

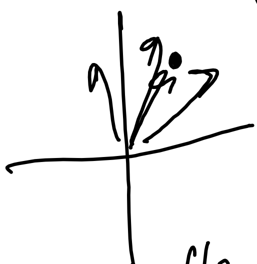
0 dim? $\boxed{\{\vec{0}\}}$

basis empty set

1 dim? $\boxed{\text{lines through the origin}}$



2 dim? $\boxed{\mathbb{R}^2}$



~~3 dim?~~

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ 90° rot ccwise

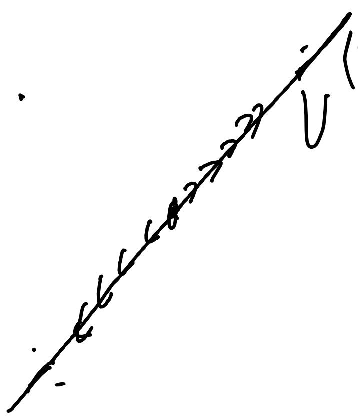
which subspace are A-invariant?

$\vec{v}$ in $V \Rightarrow A\vec{v}$ in V

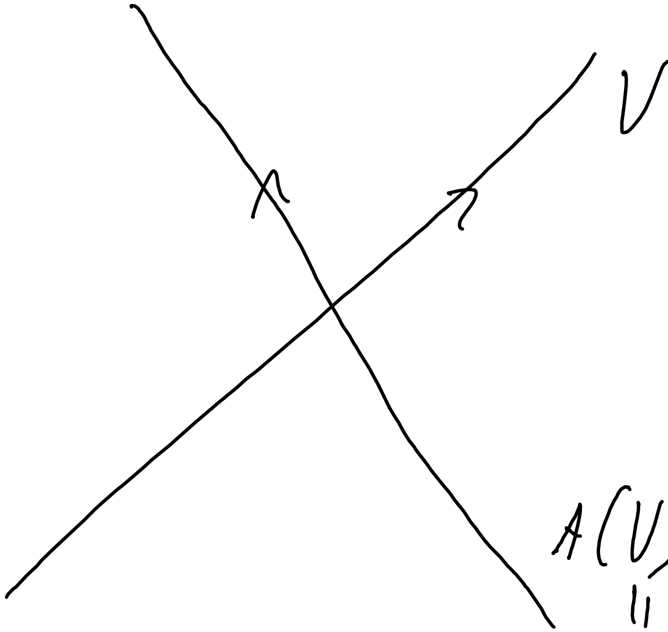
0 dim. $\{\vec{0}\}$ A-inv?

$A\vec{0} = \vec{0}$ in $\{\vec{0}\}$

1 dim'l.



V $(y=x)$, $\underline{\text{span}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$



$A(V)$

$\shortparallel$

$\{A\vec{v} \mid \vec{v} \text{ in } V\}$

$\boxed{A(V) \text{ subspace of } V}$

$\begin{pmatrix}1\\1\end{pmatrix}$, $\quad A\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix}$ <u>not</u> in V

$= \text{span}\begin{pmatrix}1\\1\end{pmatrix}$

$= \{y = x\}$

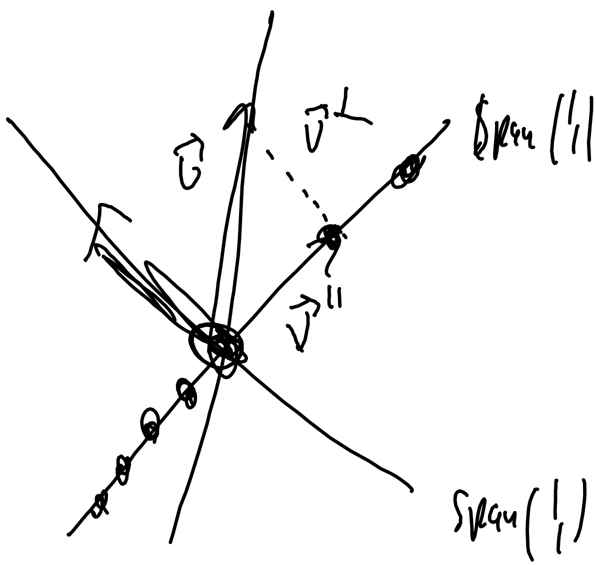
no lines A-inv

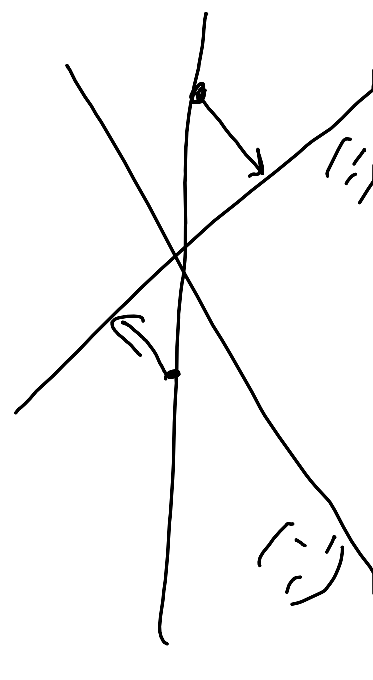
$$A = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$$

$\{\vec{0}\}$, $\mathbb{R}^2$

$\mathrm{im}(A) = \mathrm{span}\begin{pmatrix}1\\1\end{pmatrix}$ $\qquad$ $\mathrm{span}\begin{pmatrix}-1\\1\end{pmatrix} = \ker(A)$

$\{y = x\}$ $\qquad$ $\{y = -x\}$





$\dim V = 1 \quad A$ inv.

$V = \operatorname{span} \vec{v} \qquad \vec{v} \neq \vec{0}$

$A\vec{v}$ in V

$A\vec{v}$ in $\operatorname{span} \vec{v}$

$A\vec{v}$ parallel to $\vec{v}$

$A\vec{v} = \lambda \vec{v}$ some λ

scalar