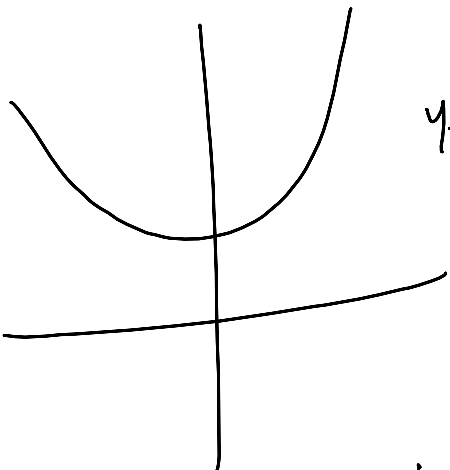
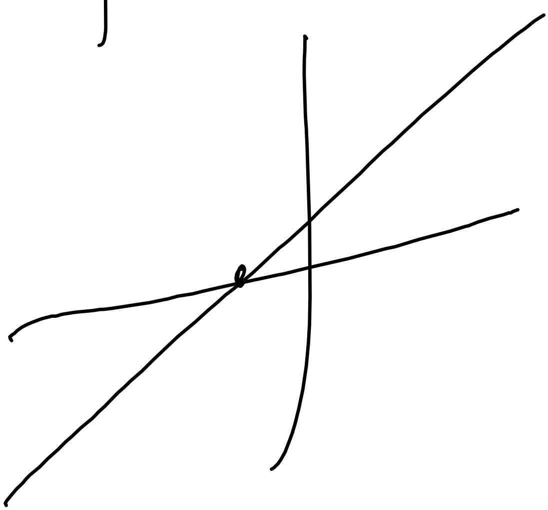
1.) Moral

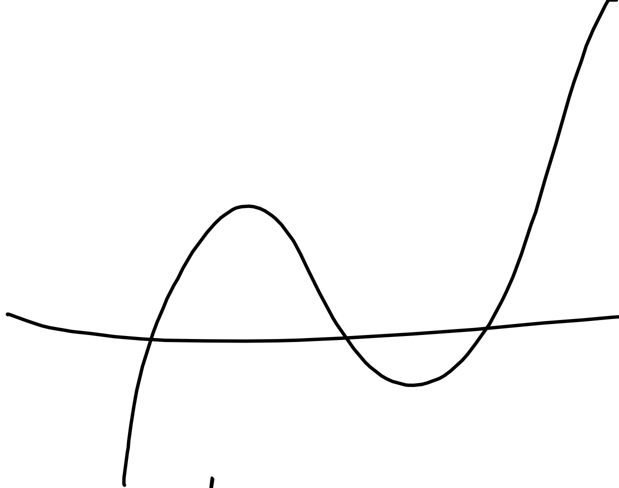
all cubics have a real root

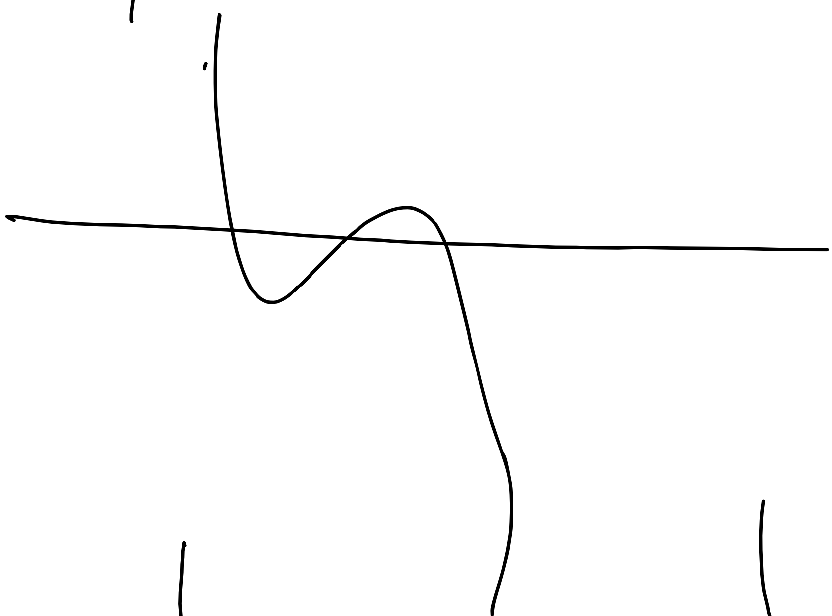


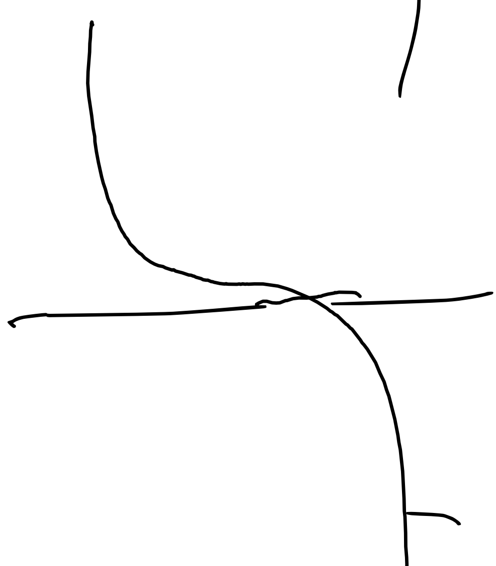
$y = x^2 + 1$

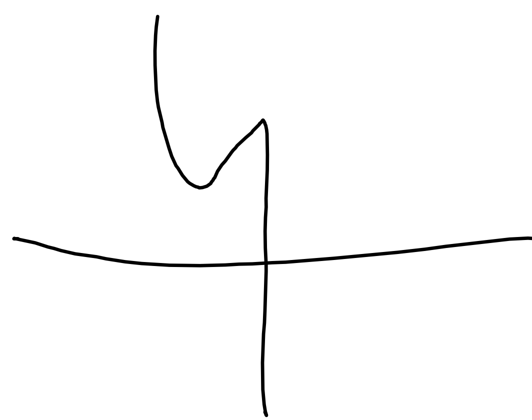
$y = x + 1$











2. $V \subseteq \mathbb{R}^n$

$A(V) \overset{\subseteq}{\cancel{\supseteq}} V$

A - inv. $\underbrace{\quad}_{\{A\vec{v} \mid \vec{v} \text{ in } V\}}$

$A\vec{v}$ in V for $\vec{v}$ in V

$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

90° ccwise rotation

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$

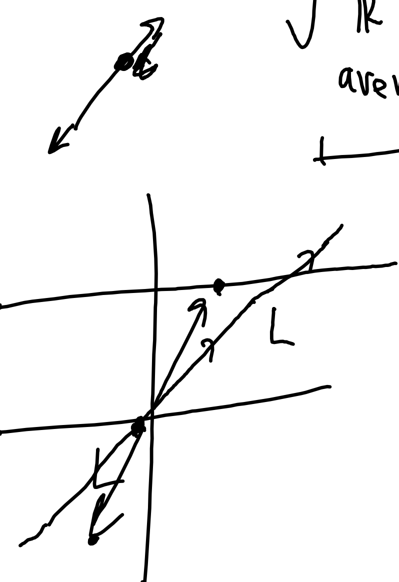
which subspaces of $\mathbb{R}^2$ - all lines through origin $\boxed{\mathbb{R}^2}$, $\boxed{\{\vec{0}\}}$

$\cancel{\{\vec{0}\}}$

✓ $\{\vec{0}\}$ are - $A\vec{0} = \vec{0}$ in $\{\vec{0}\}$

✓ $\mathbb{R}^2$ $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$ in $\mathbb{R}^2$

are not



$\mathbb{R} \not\subseteq \mathbb{R}^3$

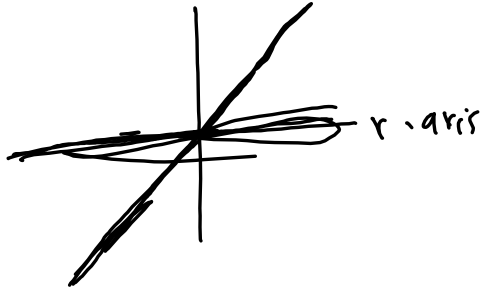
$\begin{pmatrix} 1 \end{pmatrix}, \begin{pmatrix} 5 \end{pmatrix}, ?$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 2 \end{pmatrix}, \begin{pmatrix} 17 \\ 2 \end{pmatrix}$

5 ~~in $\mathbb{R}^2$~~

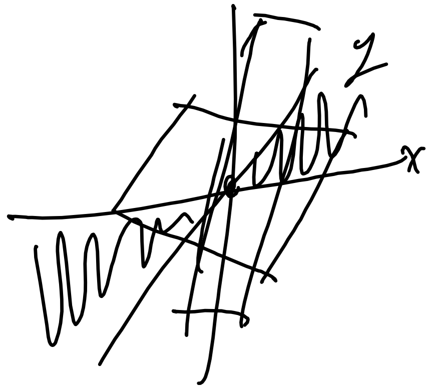
$$\mathbb{R} \hookrightarrow \mathbb{R}^3$$

$$x \longmapsto \begin{pmatrix} x \\ 0 \end{pmatrix}$$



$$\mathbb{R}^2 \hookrightarrow \mathbb{R}^3$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$



$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ - $90°$ rotation ccwise

B $n \times n$ matrix

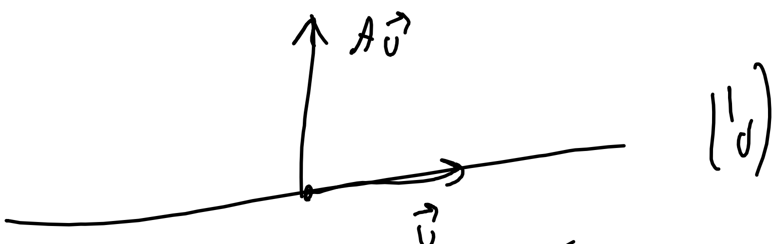
$\underline{\{\vec{0}\}}$, $\underline{\mathbb{R}^n}$ always invariant

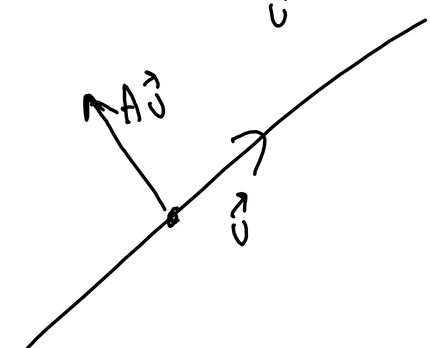
<u>are</u>

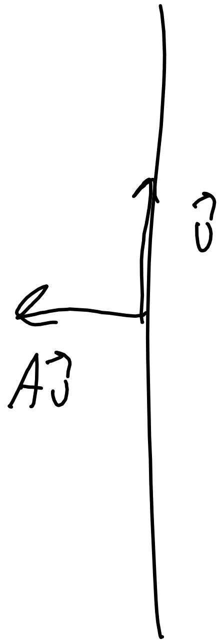
$\underline{\{\vec{0}\}}$, $\underline{\mathbb{R}^2}$

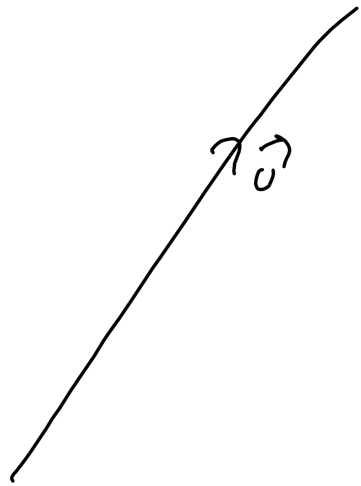
<u>are not</u>

$\vec{v}$ in L $\Rightarrow$ $A\vec{v}$ in L









$\vec{v} \neq \vec{0}$

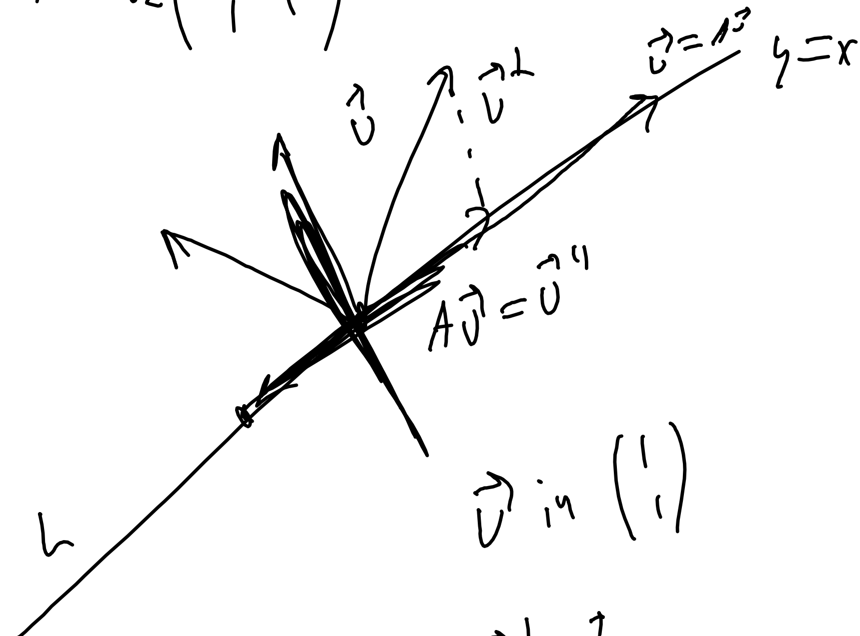
$L = \mathrm{Span}(\vec{v})$

$A\vec{v}$ perp to $\vec{v}$

$A\vec{v}$ in L? parallel to $\vec{v}$

$A\vec{0} = \vec{0}$ in L

$$A = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$



$\vec{v}$ in $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\vec{v}^{\perp} = \vec{0}$

$\vec{v}^{\parallel} = \vec{v}$

arp

- $\{\vec{0}\}$, $\mathbb{R}^2$
- $\{y=x\} = \text{span}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
- $\ker(A) = \{y=-x\} = \text{span}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $L^{\perp}$

Let $\vec{v}$ in $\ker(A)$. $A\vec{v} = \vec{0}$

$\Rightarrow$ $\vec{0}$ in $\ker(A)$

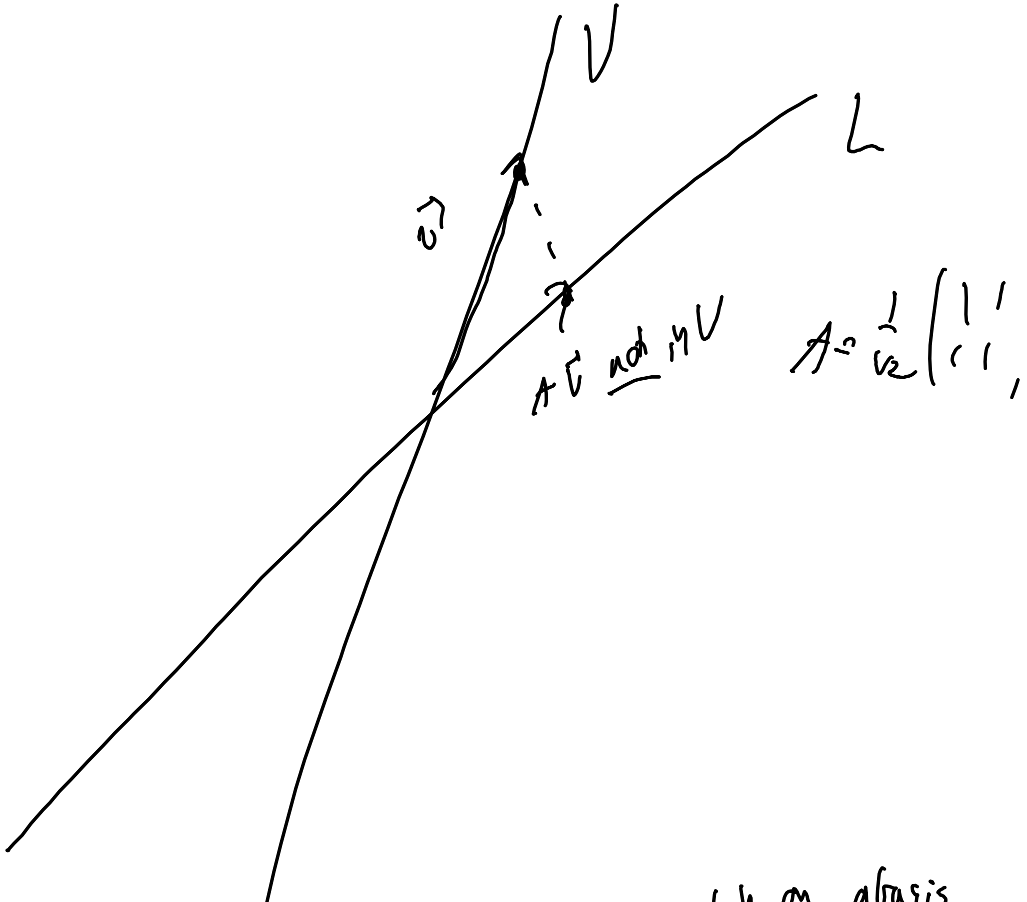
$\vec{v}$ in $\ker(A)$
$\Downarrow$
$A\vec{v}$ in $\ker(A)$

are not

everything else

V inv.

$A\vec{v}$ in V whenever $\vec{v}$ in V



To show V is inv ← suffices to check on a basis

$\vec{v}$ in V arbitrary, deduce $A\vec{v}$ in V

To show V not inv.

find a single $\vec{v}$ in V s.t. $A\vec{v}$ not in V

$L = \mathrm{span}(\vec{v})$, $\vec{v} \neq \vec{0}$

when is L A-inv?

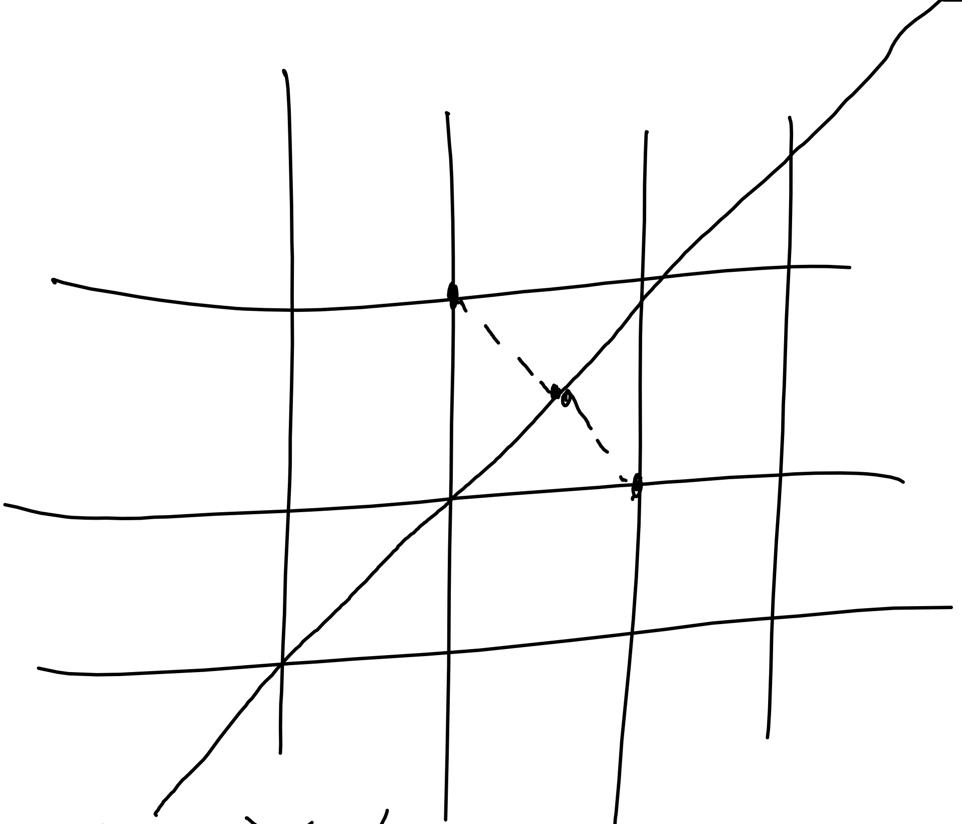
$A\vec{v}$ in $L = \mathrm{span}(\vec{v})$

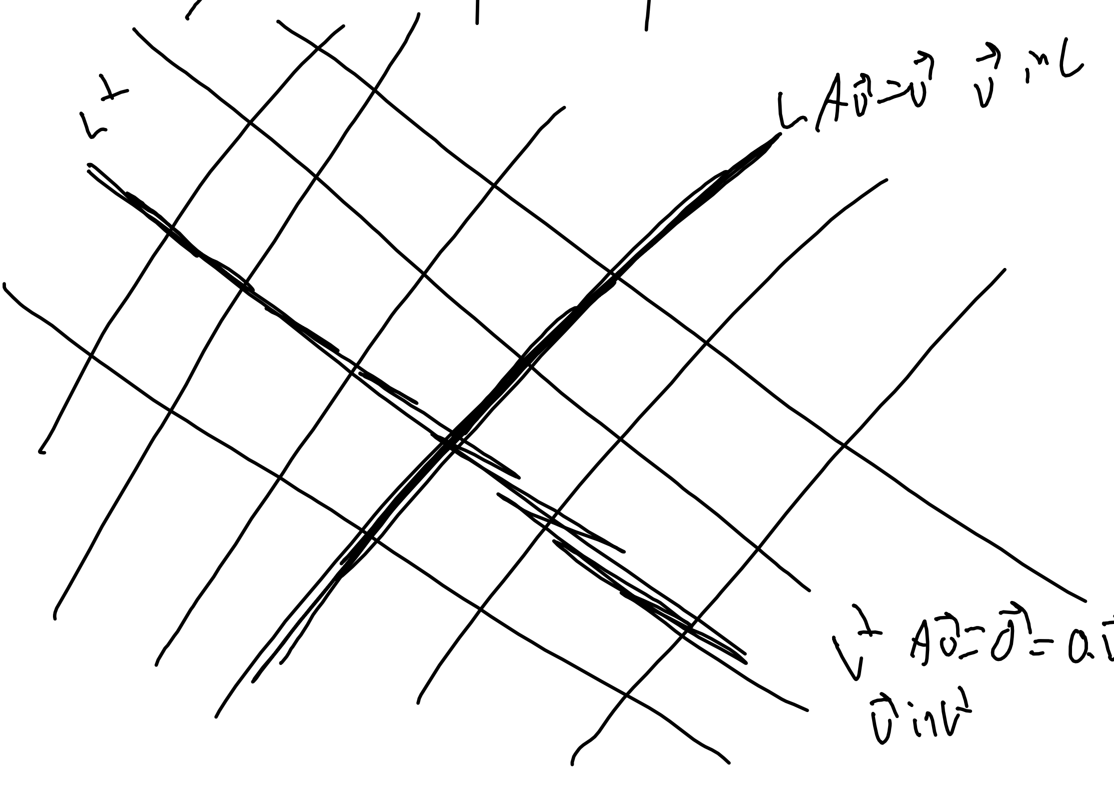
$A\vec{v} = c\,\vec{v}$ some c in $\mathbb{R}$

scalar

$\vec{v}$ "eigenvector"

c "eigenvalue"




$L^\perp$
L $A\vec{v} = \vec{v}$ $\vec{v}$ in L
$L^\perp$ $A\vec{v} = \vec{0} = 0\,\vec{v}$
$\vec{v}$ in $L^\perp$