$\vec{V}, \quad \vec{w}$

$\| \qquad \|$

$\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \quad \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$

$\left((v_1 \ldots v_n) \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix} \right)$

$\vec{V} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + \cdots + v_n w_n$

$[\ \vec{V} \cdot \vec{w} = \|\vec{V}\| \, \|\vec{w}\| \cos\theta\ ]$

length

angle between them

$[0, \pi]$

n=?

Standard basis

$\|\vec{v}\| \|\vec{w}\| \cos($ang betw $v, w)$

$+ \|\vec{v'}\| \|\vec{w}\| \cos($ang betw $v', w)$

$\vec{V} \cdot \vec{w}$

$v_1 w_1 + v_2 w_2 + \cdots + v_n w_n = \|\vec{V}\| \, \|\vec{w}\| \cos\theta$

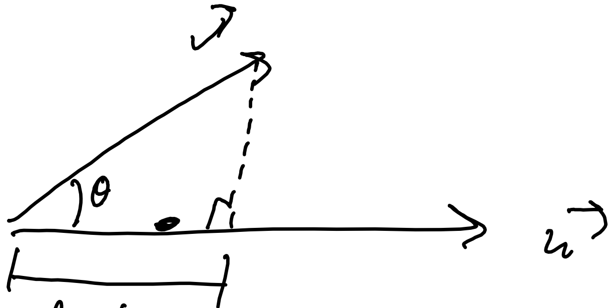
$(\vec{V} + \vec{V'}) \cdot \vec{w} = \vec{V} \cdot \vec{w} + \vec{V'} \cdot \vec{w}$

??

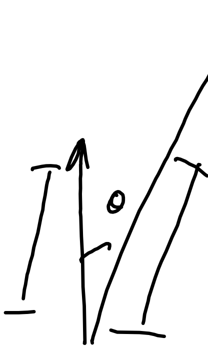
?? $\|\vec{V} + \vec{v'}\| \, \|\vec{w}\| \cos(\theta)$??

rotation invariant

$\|\vec{v}\| \, \|\vec{w}\| \cos\theta$



length $= \|\vec{v}\| \underline{\cos\theta}$



$$\vec{v}\cdot\vec{w} \overset{(?)}{=} \|\vec{v}\|\,\|\vec{w}\|\cos\theta$$

$$= D(\vec{v},\vec{w})$$

$$(c\vec{v})\cdot w = c(\vec{v}\cdot\vec{w})$$

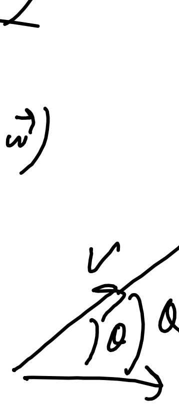
$$D(c\vec{v},\vec{w})$$

$c>0$

$$cD(\vec{v},\vec{w})$$

$c<0$

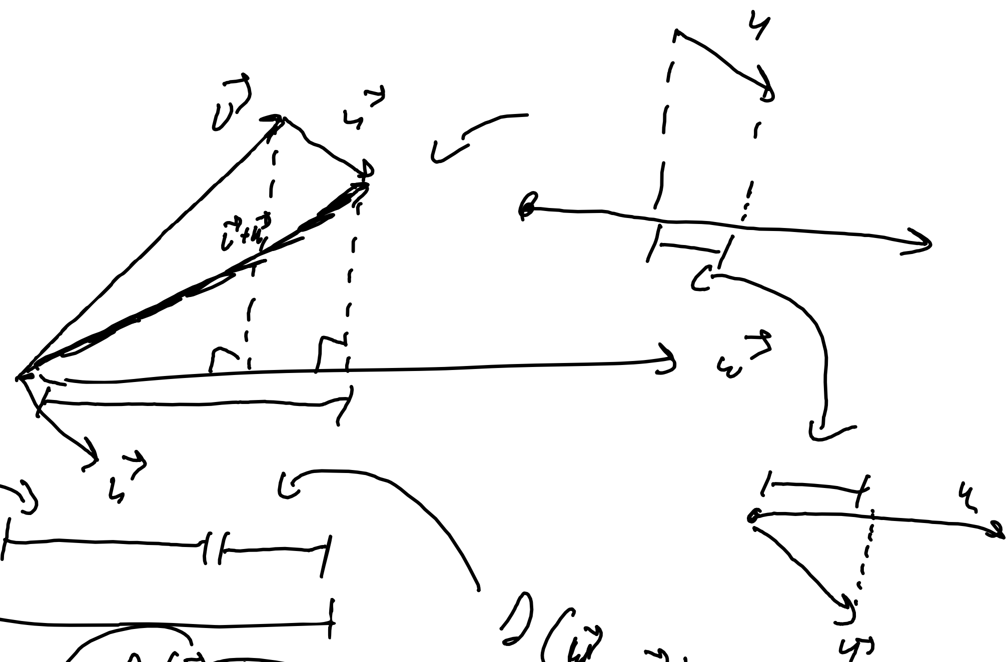
$$|c|\,\|\vec{v}\|\,\|\vec{w}\|\cos(\theta+\pi) = c\|\vec{v}\|\,\|\vec{w}\|\cos(\theta)$$



$$(\vec{v}+\vec{u})\cdot\vec{w} = \vec{v}\cdot\vec{w}+\vec{u}\cdot\vec{w}$$

$$D(c\vec{v},\vec{w}) = cD(\vec{v},\vec{w})$$

$$D(\vec{v}+\vec{u},\vec{w})$$



$$\frac{D(\vec{v}+\vec{u})}{\|\vec{w}\|}$$

$$\frac{D(\vec{u}, \vec{w})}{\|\vec{w}\|}$$

$$\|\vec{v}\|\cos\theta = \frac{D(\vec{v}, \vec{w})}{\|\vec{w}\|}$$

$$\frac{D(\vec{v}+\vec{u})}{\|\vec{w}\|} = \frac{D(\vec{u}, \vec{w})}{\|\vec{w}\|} + \frac{D(\vec{v}, \vec{w})}{\|\vec{w}\|}$$

$\vec{v}\cdot\vec{w}$

$(c\vec{v})\cdot w$

$=$

$c(\vec{v}\cdot\vec{w})$

$(\vec{v}+\vec{u})\cdot\vec{w}$

$=$

$\vec{v}\cdot\vec{w}+\vec{u}\cdot\vec{w}$

$\vec{e_1}\cdot\vec{e_2}$

$$\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}\cdot\begin{pmatrix}0\\1\\0\\\vdots\\0\end{pmatrix}$$

$=$

0

$\vec{e_1}\cdot\vec{e_1}=1$

$$\begin{pmatrix}1\\0\\0\\\vdots\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}$$

$=1\cdot1+0\cdot0+0\cdot0\ldots+0\cdot0$

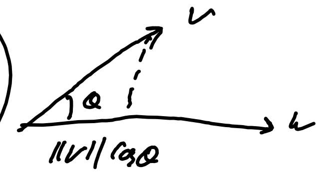
$=1$

$D(\vec{v},\vec{w})=\|\vec{v}\|\|\vec{w}\|\cos\theta$

$D(c\vec{v},\vec{w})$

$=$

$cD(\vec{v},\vec{w})$

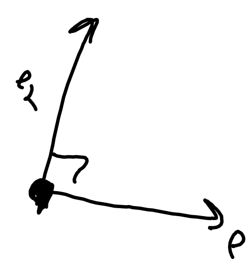


$D(\vec{v}+\vec{u},\vec{w})$

$=$

$D(\vec{v},\vec{w})+D(\vec{u},\vec{w})$

$D(\vec{e_1},\vec{e_2})=0$



$D(\vec{e_1},\vec{e_1})=1$

$$D(\vec{e_i}, \vec{e_j}) = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

$$\vec{e_i} \cdot \vec{e_j} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

$\|e_i\| = 1$

perpindicular

$$\vec{v} = v_1\vec{e_1} + \cdots + v_n\vec{e_n}$$

$$\vec{w} = w_1\vec{e_1} + \cdots + w_n\vec{e_n}$$

$$D(\vec{v}, \vec{w}) = D(v_1\vec{e_1} + \cdots + v_n\vec{e_n},\ w_1\vec{e_1} + \cdots + w_n\vec{e_n})$$

$\| \vec{v} \| \| \vec{w} \| \cos\theta$

$$\overset{(!)}{=} \sum_i \sum_j v_i w_j D(\vec{e_i}, \vec{e_j})$$

$$D\left(\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}3\\2\end{pmatrix}\right) = D(2\vec{e_1} + \vec{e_2},\ 3\vec{e_1} + 2\vec{e_2})$$

$$= D(2\vec{e_1},\ 3\vec{e_1} + 2\vec{e_2}) + D(\vec{e_2},\ 3\vec{e_1} + 2\vec{e_2})$$

$$= 2D(\vec{e_1},\ 3\vec{e_1} + 2\vec{e_2}) + D(\vec{e_2},\ 3\vec{e_1} + 2\vec{e_2})$$

$$= 2D(\vec{e_1},\ 3\vec{e_1}) + 2D(\vec{e_1},\ 2\vec{e_2}) + D(\vec{e_2},\ 3\vec{e_1}) + D(\vec{e_2},\ 2\vec{e_2})$$

6

0

0

2

$6+2=8$

$$\begin{pmatrix}2\\1\end{pmatrix}\cdot\begin{pmatrix}3\\2\end{pmatrix} = 2\cdot 3 + 1\cdot 2$$

$$\underbrace{2D(e_1, 3e_2)}_{2\cdot 3} + \cancel{2D(e_1, 2e_2)}^{0} + \cancel{D(e_2, 3e_1)}_{0} + \underbrace{D(e_2, 2e_2)}_{1\cdot 2}$$

$\vec{e_i}$ w/ $\vec{u_i}$

$\vec{u_i}\cdot\vec{u_j} = \begin{cases}1 & i=j\\ 0 & \text{o/w}\end{cases}$

$\vec{v}\cdot\vec{w} = \|\vec{v}\|\|\vec{w}\|\cos\theta$

Both sides have distributivity

agree on standard basis vectors

$T(e_i) = S(e_i)$ for all i

$\Downarrow$

$T = S$

wksht 5

1. double (calculus)]

Rest. challenge]

1. $\begin{pmatrix} v_1 \\ \vdots \\ v_4 \end{pmatrix} \rightsquigarrow (x_1, x_2, x_3, \ldots \ldots)$

$\vec{v} \cdot \vec{w}$

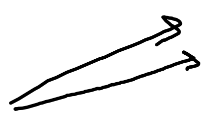
$\parallel$

$v_1 w_1 + \dots + v_4 w_4$

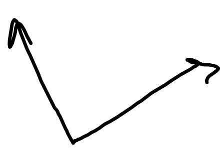
$\underline{\vec{x} \cdot \vec{y}} = \underbrace{x_1 y_1 + x_2 y_2 + \dots}$

$(1, 0, 0, \ldots)$

$u\,(1, 1/2, 1/4, 1/8, 1/16, \ldots)$



cos, sin



a) $\vec{v} = (1, 1/2, 1/4, 1/8, \ldots)$

$\|\vec{v}\| \overset{\text{def}}{=} \sqrt{v_1^2 + v_2^2 + \dots} = \sqrt{\underbrace{1^2 + 2^{-2} + 2^{-4} + 2^{-6} \dots}}$

$= \sqrt{\frac{1}{1 - \frac{1}{4}}} = \frac{2}{\sqrt{3}}$

$$\vec{v} = \left(1, 1/2, 1/4, 1/8, 1/16, \ldots\right)$$

$$v_i = 2^{-i}$$

$$v_i^2 = 2^{-2i} = \left(\frac{1}{4}\right)^i$$

$$\sqrt{\sum_{i=0}^{\infty} v_i^2} = \sqrt{\sum_{i=0}^{\infty} 2^{-2i}}$$

$$= \sqrt{\frac{1}{1-\frac{1}{4}}}$$

$$= \sqrt{\frac{1}{\left(\frac{3}{4}\right)}}$$

$$= \sqrt{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}}$$

3, 4, 19

$\vec{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\underline{\vec{v_1}} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\underline{\vec{v_2}} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\beta = (\vec{v}_1, \vec{v}_1)$

(why a basis? not scalars)

$\boxed{T(\vec{x}) = A\vec{x}}$

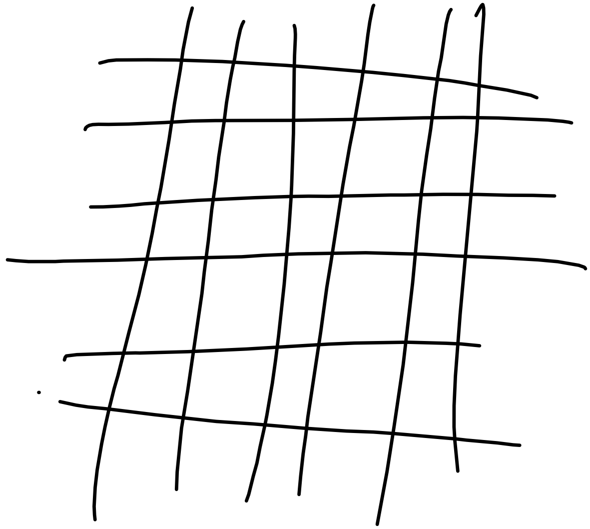
$\underline{[T]_\beta = ?}$,

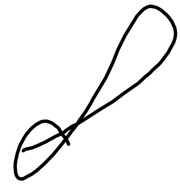
$T(\vec{x}) = A\vec{x}$, Let $[T]_\delta = A$

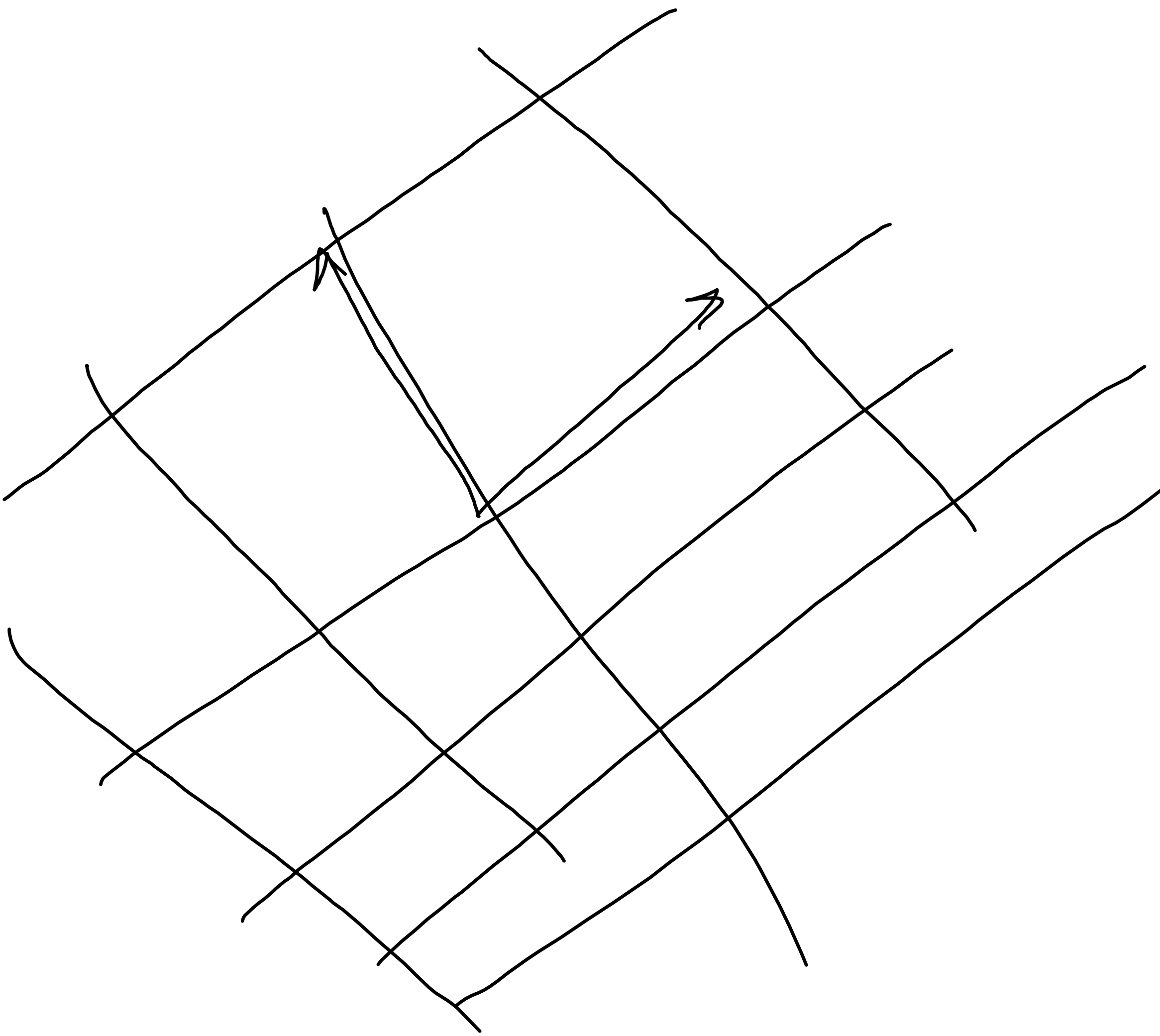
expressed in the standard basis

$= [T]_\delta [x]_\delta$

δ standard basis







$$[T(\vec{x})]_\alpha = [T]_\alpha \, [\vec{x}]_\alpha$$

$$[T(\vec{x})]_\beta = [T]_\beta \, [\vec{x}]_\beta$$

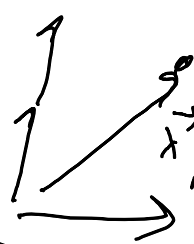
?

$\begin{pmatrix}1\\d\end{pmatrix}$ $\begin{pmatrix}1\\2\end{pmatrix} = [\vec{x}]_\alpha$

$\vec{x} = 1\,\vec{e_1} + 2\,\vec{e_2}$

$T(\vec{x})$

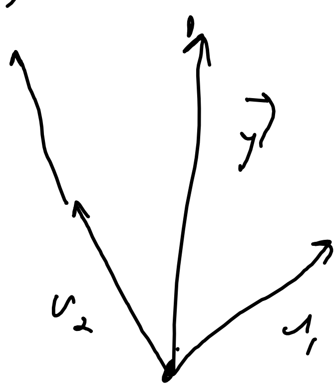
?? $\begin{pmatrix}a\\b\end{pmatrix}_\beta$



$\begin{pmatrix}1\\2\end{pmatrix} = [\vec{y}]_\beta$

$\vec{y} = 1\cdot\vec{v_1} + 2\vec{v_2}$

$=$



$$\underline{[T(\vec v)]_\beta} = \underbrace{[T]_\beta}\ [\vec x]_\beta$$

1. "$S^{-1}AS$" method ✓

2. comm. diagram method ?

3. column by column method ?

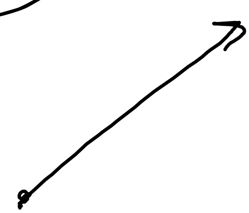
$$S = \begin{pmatrix} | & | \\ \vec v_1' & \vec v_2' \\ | & | \end{pmatrix} \qquad \beta = (\vec v_1, \vec v_2)$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

$$S\,\underline{[\vec x]_\beta} = \underline{[\vec x]_\alpha} \rightarrow \begin{pmatrix} ? \end{pmatrix}$$

$$[\vec x]_\beta = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\vec x = \vec v_1$$



$$S\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ 1 \end{pmatrix}} = [\vec v_1]_\alpha$$

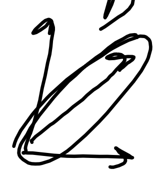
$$S^{-1}AS = [T]_\beta$$

$$A = [T]_\mathcal{E}$$

$$S = {}_\mathcal{E}[I]_\beta$$

$$({}_\mathcal{E}[I]_\beta)^{-1}\,{}_\mathcal{E}[T]_\mathcal{E}\,{}_\mathcal{E}[I]_\beta = {}_\beta[T]_\beta$$

$$[T]_\beta [x]_\beta = [T(\vec{x})]_\beta$$



$$\begin{pmatrix}1\\2\end{pmatrix}$$

$$S^{-1}AS[x]_\beta$$

$$1\begin{pmatrix}1\\0\end{pmatrix}+2\begin{pmatrix}0\\1\end{pmatrix}$$

$$S^{-1}A[x]_\mathcal{E}$$

$$[T(\vec{x})]_\mathcal{E} = A[x]_\mathcal{E}$$

$$T(\vec{x}) = A\vec{x}$$

$$[T(\vec{x})]_\mathcal{E}$$

$$S^{-1}[T(\vec{x})]_\mathcal{E} = [T(\vec{x})]_\beta$$

$$\begin{pmatrix}1&1\\1&-1\end{pmatrix}^{-1}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix} = [T]_{\mathcal{B}}$$

$$[T(\vec{x})]_{\mathcal{B}} = [T]_{\mathcal{B}}\,[\vec{x}]_{\mathcal{B}}$$

$[T]_{\mathcal{E}}$

$$\begin{tikzcd}
\text{output } [x]_{\mathcal{E}} \arrow[r, mapsto, "A"] & {[T(\vec{x})]_{\mathcal{E}}} \\
\text{input } [x]_{\mathcal{B}} \arrow[u, mapsto, "S\ ??"] \arrow[r, dashed, mapsto, "?"'] \arrow[ur, bend left, "?"] & {[T(\vec{x})]_{\mathcal{B}}} \arrow[u, mapsto, "S\ ??"']
\end{tikzcd}$$

$B = \underline{[T]_{\mathcal{B}}}$

$\underline{AS}\,[\vec{x}]_{\mathcal{B}} = \underline{SB}\,[\vec{x}]_{\mathcal{B}}$ for all vectors $\vec{x}$

$\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$

$AS = SB$

$\underline{B = S^{-1}AS}$

$$\boxed{S}\,[x]_{\mathcal{B}} = [x]_{\mathcal{A}}$$

$$\mathcal{B} = (\vec{v}_1, \vec{v}_2) \xrightarrow{\vec{x}} \begin{pmatrix}1\\0\end{pmatrix}_{\mathcal{B}}$$

$$\xrightarrow{S\vec{v}} \begin{pmatrix}1\\1\end{pmatrix}_{\mathcal{A}}$$

$$[\vec{v}_1]_{\mathcal{B}} = \begin{pmatrix}1\\0\end{pmatrix} \qquad [\vec{v}_2]_{\mathcal{B}} = \begin{pmatrix}0\\1\end{pmatrix}$$

$$\vec{v}_1 \overset{\checkmark}{=} 1\cdot\vec{v}_1 + 0\cdot\vec{v}_2 \qquad \vec{v}_2 = 0\cdot\vec{v}_1 + 1\cdot\vec{v}_2$$

$$\vec{x} = \underline{a}\,\vec{v}_1 + \underline{b}\,\vec{v}_2$$

$$[\vec{x}]_{\mathcal{B}} = \begin{pmatrix}a\\b\end{pmatrix}$$

$$S[v_1]_\beta = [v_1]_\alpha \qquad S = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & b \\ 1 & d \end{pmatrix}$$

$$S[v_2]_\beta = [v_2]_\alpha$$

$$\begin{pmatrix} b \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} v_1' & v_2' \end{pmatrix}$$

$$
\begin{tikzcd}
{[v]_\alpha} \arrow[r, mapsto, "A"] & {[T(\vec{v})]_\alpha} \\
{[v]_\beta} \arrow[u, mapsto, "S"] \arrow[r, mapsto, "?"'] & {[T(\vec{v})]_\beta} \arrow[u, mapsto, "S"']
\end{tikzcd}
$$

$$B = S^{-1}AS$$

or

$$= SAS^{-1}$$

$$
\begin{tikzcd}
\beta \arrow[r] \arrow[d] & \beta \\
\alpha \arrow[r] & \alpha \arrow[u]
\end{tikzcd}
$$

$$ {}_\beta[T]_\beta = {}_\beta[I]_\alpha \; {}_\alpha[T]_\alpha \; {}_\alpha[I]_\beta $$

38. Find β of $\mathbb{R}^n$

s.t. $[T]_\beta$ diagonal

$T(\vec{x}) =$ refl along $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ in $\mathbb{R}^2$

---

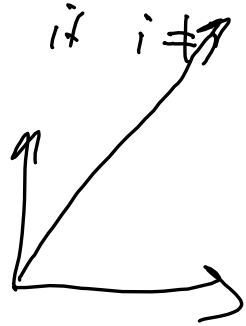
diagonal matrix?

$$\begin{pmatrix} * & & & 0 \\ & * & & \\ & & \ddots & \\ 0 & & & * \end{pmatrix}$$

$$\beta_{ij} = 0 \quad \text{if } i \neq j$$

$T\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right)$

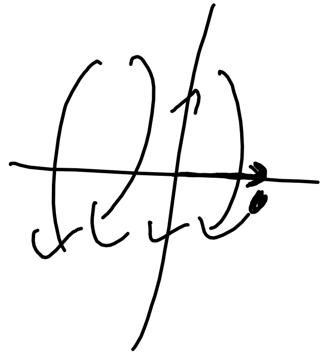
$T\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)$



$U : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

reflection about x-axis

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$



$A = [U]_{\mathcal{B}} = ?$

$U(\vec{x}) = A\vec{x}$

$U(\vec{e_1}) = \vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. $U(\vec{e_1}) = A\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

first col of A

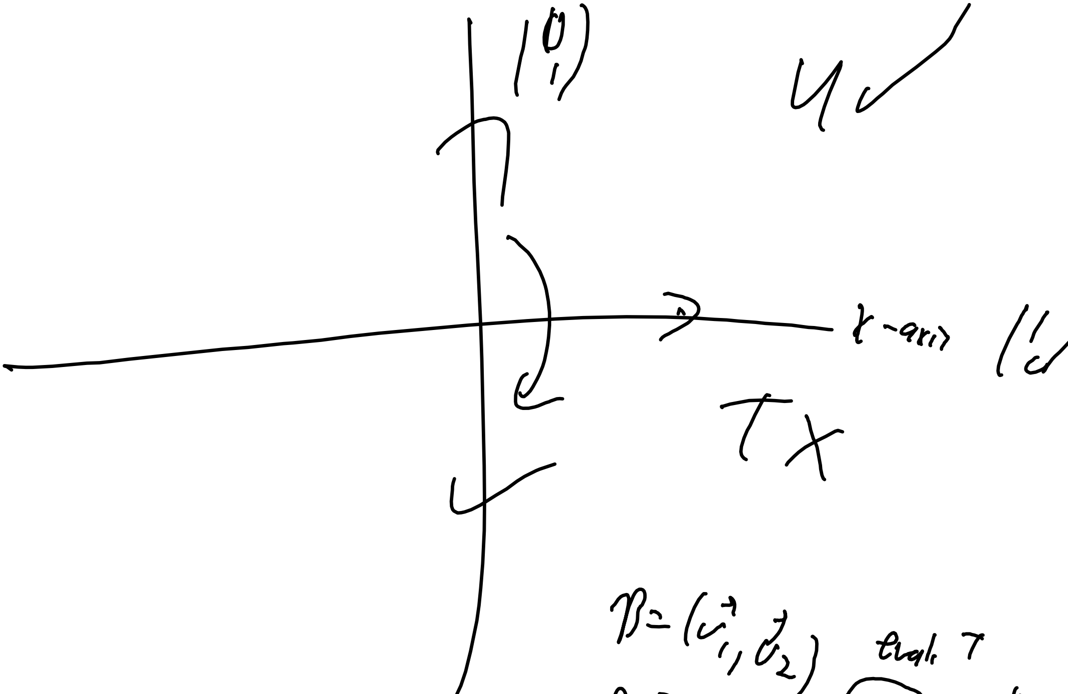
$U(\vec{e_2}) = -\vec{e_2} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

$U\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

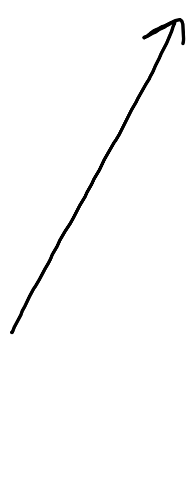
$$[U]_{\mathcal{B}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

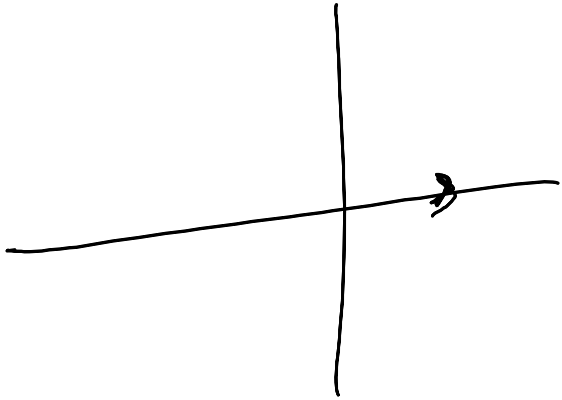


$\beta = (\vec{v}_1, \vec{v}_2)$ eval. T

$[T]_\beta = \left[[T(v_1)]_\beta \quad [T(v_2)]_\beta \right]$ writing language

T refl about $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$





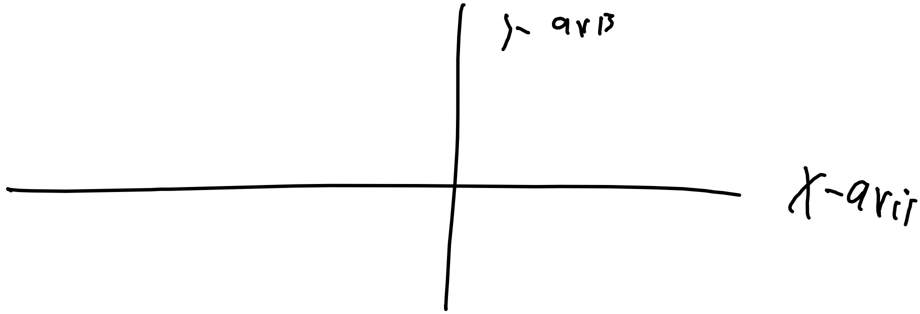
U ✓

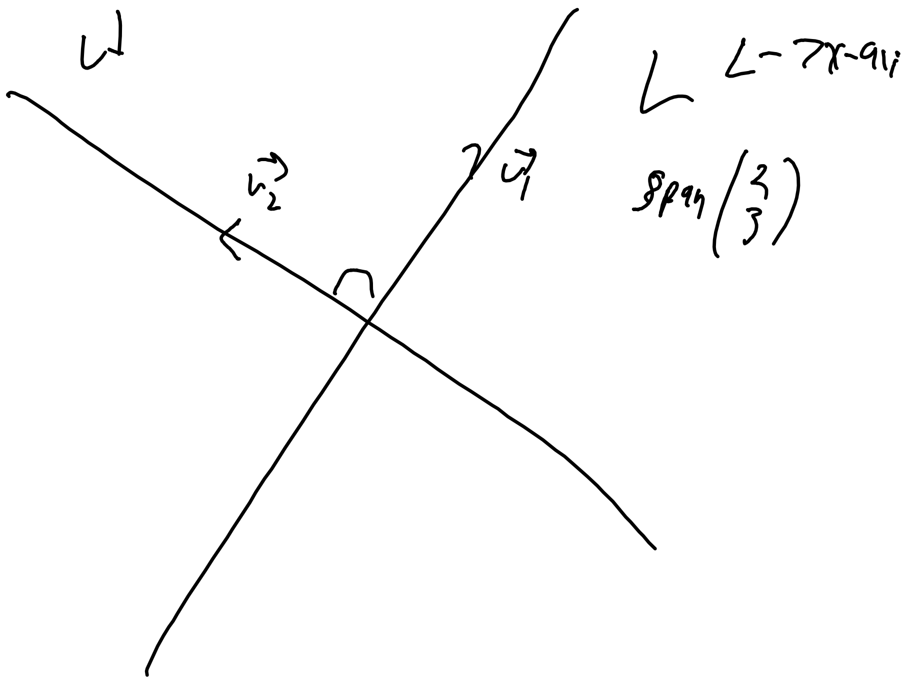
why is $\mathcal{B}$ good for U?

$\vec{e}_1$ spans x-axis

U defined in terms of x-axis

$\vec{e}_2$ spans y-axis





U	T
x-axis	$L = \text{span}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
y-axis	$L^{\perp} = \text{span}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$$\vec{v}_1 = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$\vec{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\mathcal{B} = (\vec{v}_1, \vec{v}_2)$$

$$T(\vec{v}_1) = \vec{v}_1$$

$$T(\vec{v}_2) = -\vec{v}_2$$

$$[\vec{v}_1]_{\mathcal{B}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$[-\vec{v}_2]_{\mathcal{B}} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$B = [T]_{\mathcal{B}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S^{-1} A S = B$$

$$A = S B S^{-1}$$

$U(\vec{e_1}) = \vec{e_1}$

$U(\vec{e_2}) = -\vec{e_2}$

refl X-axis

$U(\vec{e_i})$ parallel to $\vec{e_i}$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$T(\vec{v_1}) = \vec{v_1}$

$T(\vec{v_2}) = -\vec{v_2}$

$T(\vec{v_i})$ parallel to $\vec{v_i}$

$$D = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$

$$\begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}$$

$De_1 = a_1 e_1$

$De_2 = a_2 e_2$

$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\mathcal{L}^n$

$$T \rightsquigarrow [T]_{\mathcal{B}}$$

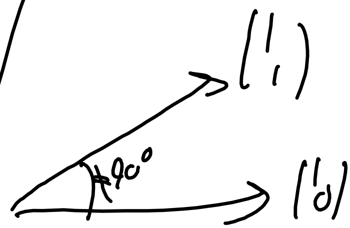
$$\leftsquigarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{\mathcal{B}}$$

$$T(\vec{v_1}) = \vec{v_1}$$
$$T(\vec{v_2}) = -\vec{v_2}$$

$$\left(T(\vec{v}) = c\vec{v} \right)$$

$$\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \frac{1}{\sqrt{5}}\begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Orthonormal



$$T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$$

$$T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\beta = \left\{\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}\right\}$$

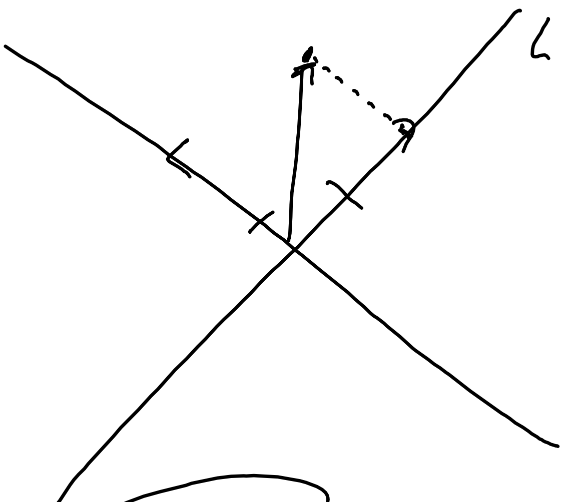
$$[T]_\beta = \begin{pmatrix}2 & 0\\0 & 1\end{pmatrix}$$

$T(\vec{x}) = \overset{\text{orthog.}}{\text{proj}}$ onto L spanned by $\begin{pmatrix}2\\3\end{pmatrix}$

$\begin{pmatrix}-3\\2\end{pmatrix}$, $\begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}-1\\5\end{pmatrix}$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} = [T]_{\mathcal{B}}$$

$$T\left(\begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}-3\\2\end{pmatrix}\right) = \underbrace{T\begin{pmatrix}2\\3\end{pmatrix}}_{=\begin{pmatrix}2\\3\end{pmatrix}} + \underbrace{T\begin{pmatrix}-3\\2\end{pmatrix}}_{=0}$$



$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mathcal{B} = \left(\underbrace{\begin{pmatrix}2\\3\end{pmatrix}\frac{1}{\sqrt{13}}}_{=v_1}, \underbrace{\frac{1}{\sqrt{13}}\begin{pmatrix}-3\\2\end{pmatrix}}_{=v_2} \right)$$

$T(\vec{v}_1) = \vec{v}_1$, $\begin{pmatrix}1\\0\end{pmatrix}$

$T(\vec{v}_2) = \vec{0}$, $\begin{pmatrix}0\\0\end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$U(\vec{x}) =$ proj onto x-axis

$$\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right]$$

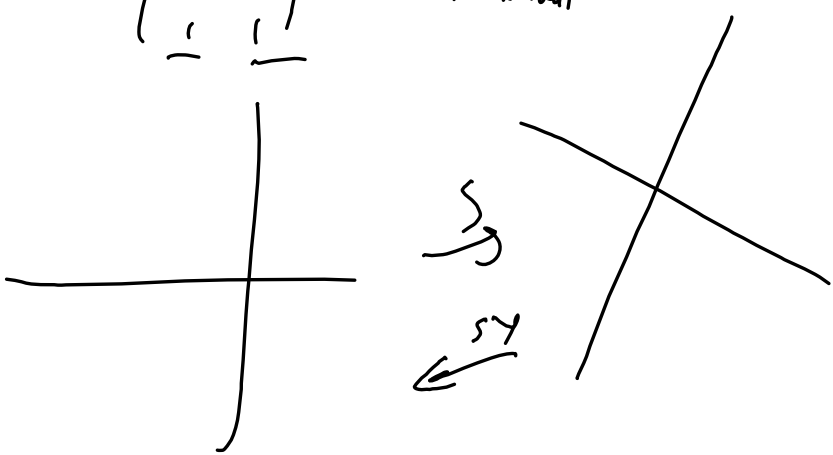
$$U\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$$

$$S[T]_{\mathcal{B}} S^{-1} = [T]_{\alpha}$$

→ columns basis

↑ orthonormal

$$S = \begin{pmatrix} | & | \\ v_1 & v_2 \\ | & | \end{pmatrix}$$



3.4.19.)

$B = S^{-1} A S$

$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

bashit out

$T(\vec{e}_1) = \vec{e}_2$

$T(\vec{e}_2) = \vec{e}_1$

$S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

cols are $\mathcal{B}$

$\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \vec{e}_1\right)$

A?

matrix for T w.r.t. standard basis $\mathcal{S}$

B? $AS = SB$

matrix for T wrt basis $\mathcal{B}$

$T(\vec{x}) = A\vec{x}$

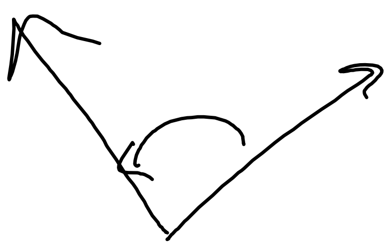
$\begin{pmatrix}1\\0\end{pmatrix}$ in $\mathcal{E}$

$\begin{pmatrix}1\\0\end{pmatrix}$

$[T(\vec{v})]_{\mathcal{E}} = A\,[x]_{\mathcal{E}}$

$[T(\vec{x})]_{\mathcal{B}} = B\,[x]_{\mathcal{B}}$

$\mathcal{B} = (\vec{v}_1, \vec{v}_2)$



$\vec{x}$

"

$[\vec{x}]_{\mathcal{E}} = \begin{pmatrix}1\\3\end{pmatrix}$

$1\vec{e}_1 + 3\vec{e}_2$

$[\vec{x}]_{\mathcal{B}} = \begin{pmatrix}1\\3\end{pmatrix} = \underbrace{1}\vec{v}_1 + \underbrace{3}\vec{v}_2$

Let $[\vec{y}]_{\mathcal{B}} = \begin{pmatrix}1\\0\end{pmatrix}$

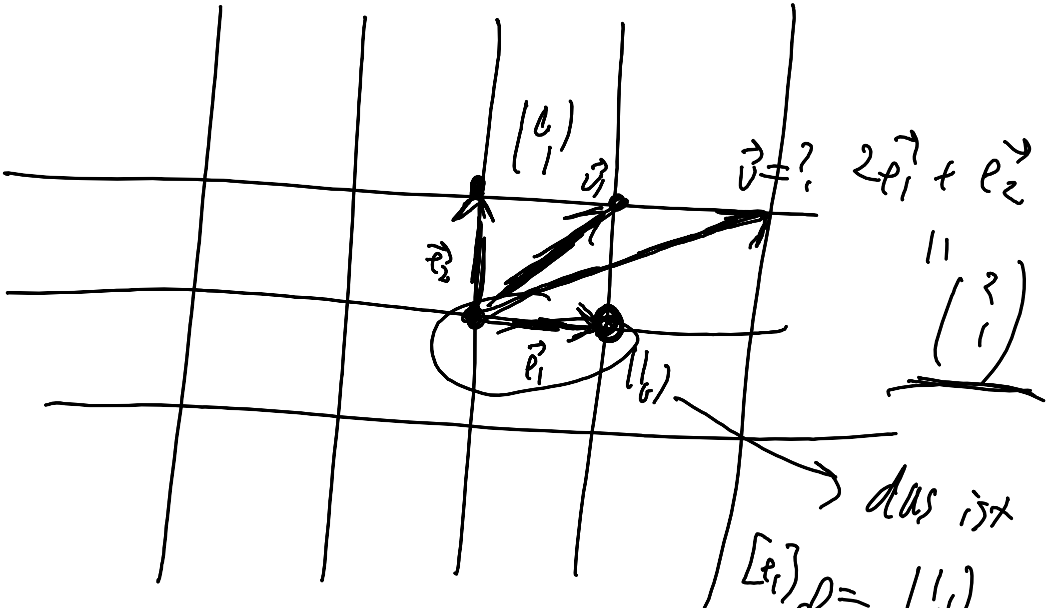
def. $\vec{y} = 1\cdot\vec{v}_1 + 0\cdot\vec{v}_2$

$= \vec{v}_1$

$[\vec{v}_1]_{\mathcal{B}} = \begin{pmatrix}1\\0\end{pmatrix}$

$[\vec{v}_1]_{\mathcal{E}} = \begin{pmatrix}1\\1\end{pmatrix}$, $[\vec{v}_2]_{\mathcal{E}} = \begin{pmatrix}1\\-1\end{pmatrix}$

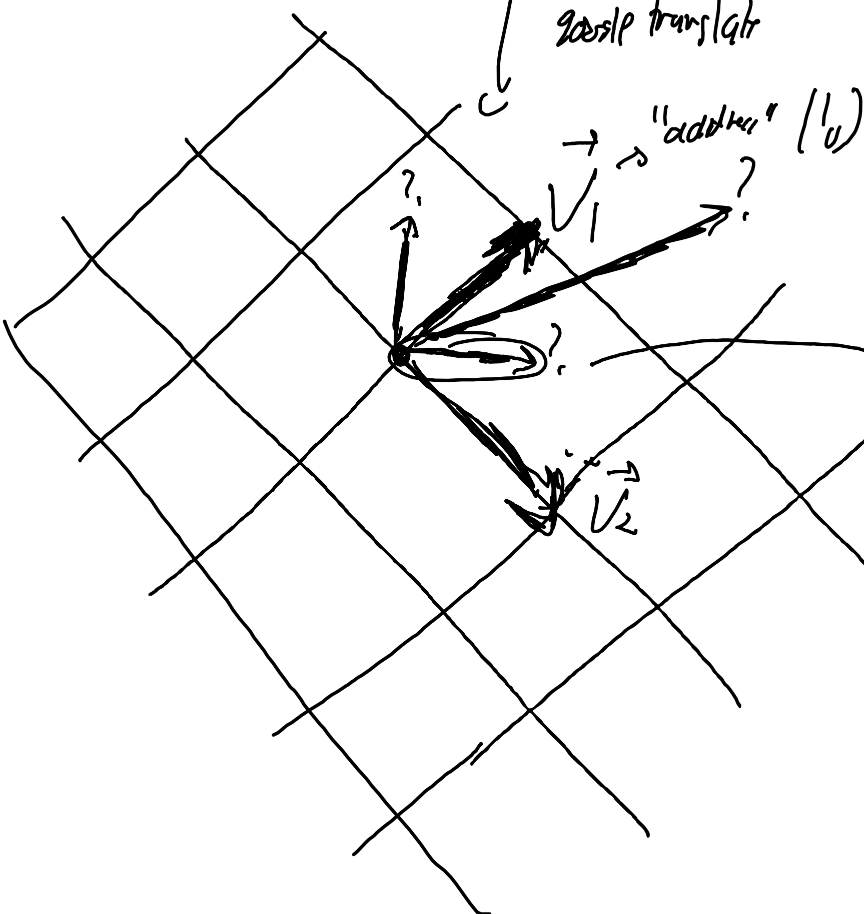
$\vec{v}_1$ $\vec{v}_2$



das ist

$[e_1]_\alpha = \begin{pmatrix}1\\0\end{pmatrix}$

google translate



this is $\begin{pmatrix}\frac{1}{2}\\-\frac{1}{2}\end{pmatrix} = [e_1]_\beta$

$$\begin{array}{ccc}
[x]_{\mathcal{S}} & \xmapsto{\ A\ } & [T(\vec{x})]_{\mathcal{S}} \\
\Big\uparrow S & & \\
[x]_{\mathcal{B}} & \xmapsto{\ B\,=\,?\ } & [T(\vec{x})]_{\mathcal{B}} \\
\text{input} & & \text{output}
\end{array}$$

$[\vec{x}]_{\mathcal{B}} = \begin{pmatrix} a \\ b \end{pmatrix}$

$\Updownarrow$

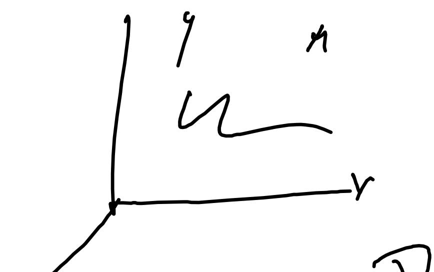
$\vec{x} = a\vec{v}_1 + b\vec{v}_2$

$S[x]_{\mathcal{B}} = [x]_{\mathcal{S}}$

$[\vec{v}_1]_{\mathcal{B}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\Updownarrow$

$\vec{v}_1 = \underline{1}\cdot\vec{v}_1 + \underline{0}\cdot\vec{v}_2$



$[\vec{v}_2]_{\mathcal{B}} = \begin{pmatrix} a \\ b \end{pmatrix}$

$a\vec{e}_1 + b\vec{e}_2 = \vec{e}_2$

$a=0$
$b=1$
$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$a\vec{v}_1 + b\vec{v}_2 = \vec{v}_2$

$a\begin{pmatrix} 1 \\ 1 \end{pmatrix} + b\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$a+b=1$, $b=1-a$

$a-b=-1$, $a-(1-a)=-1$

$2a-1=-1$, $a=0$, $b=1$

$a=0$
$b=1$

$[v_2]_{\mathcal{B}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\beta = (\vec{u_1}, \vec{u_2}, \ldots, \vec{u_n})$$

check: $[u_i]_\beta = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \leftarrow i^{th}$ place

$$\begin{tikzcd}
{[x]_\alpha} \arrow[r, mapsto, "A"] & {[T(\vec{x})]_\alpha} \\
{[x]_\beta} \arrow[u, mapsto, "S"] \arrow[r, mapsto, "B"] \arrow[ur, bend left] & {[T(\vec{x})]_\beta} \arrow[u, mapsto, "S"']
\end{tikzcd}$$

? ?

? $\rightarrow$ $AS[x]_\beta = SB[x]_\beta$ $\rightarrow$?

$$AS = SB$$

$S[x]_\beta = [x]_\alpha$, $\lambda = v_1$

$S\underbrace{[\vec{v_1}]_\beta} = [v_1]_\alpha = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$S\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $S\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $S = \begin{pmatrix} 1 & * \\ 1 & * \end{pmatrix}$

$$S\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix}$$

$$\|$$

$$S\underbrace{[\vec{v}_2]_\beta}_{\begin{pmatrix}0\\1\end{pmatrix}} = [\vec{v}_2]_\alpha$$

$$S = \begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$$

B ?

$[\vec{e}_1]_{\mathcal{E}} = \begin{pmatrix}1\\0\end{pmatrix}$

$B\begin{pmatrix}r\\s\end{pmatrix}$ $\quad B[\vec{x}]_{\mathcal{B}} = [T(\vec{x})]_{\mathcal{B}}$ $(*)$

$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix}1\\0\end{pmatrix}$ $\mathcal{B}$

translat

$\begin{pmatrix}1\\0\end{pmatrix}$ $\mathcal{E}$

assertion

$1\vec{v}_1 + 0\vec{v}_2 = \vec{v}_1$

$B\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}a\\c\end{pmatrix}$

$B\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}b\\d\end{pmatrix}$

$\begin{pmatrix}1\\0\end{pmatrix} = [\vec{v}_1]_{\mathcal{B}}$, $\quad \begin{pmatrix}0\\1\end{pmatrix} = [\vec{v}_2]_{\mathcal{B}}$

$\begin{pmatrix}a\\c\end{pmatrix} = B[\vec{v}_1]_{\mathcal{B}} = [T(\vec{v}_1)]_{\mathcal{B}}$

want to know

how we'll find it

first, $T(\vec{v}_1)$ ✓

Second, express that in $\mathcal{B}$.

$[\vec{x}]_{\mathcal{B}} = \begin{pmatrix}r\\s\end{pmatrix}$

def

$\vec{x} = r\vec{v}_1 + s\vec{v}_2$

$T(\vec{v_1})$?

$T(\vec{x}) = A\vec{x}$

$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\vec{u_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\shortparallel$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

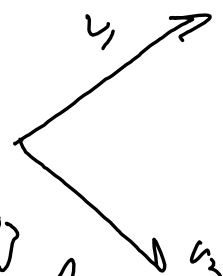
$$\shortparallel$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$[\vec{x}]_{\ell} = \vec{x}$

$\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]_{\ell} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$T(\vec{x}) = A\vec{x}$



$[T(\vec{x})]_{\ell} = A\,[\vec{x}]_{\ell}$

(M_{ℓ})

$\left(A = {}_{\underline{\ell}}[T]_{\underline{\ell}} \right)$

$v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$[\vec{v_1}]_{\ell} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$[\vec{v_2}]_{\ell} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$T(\vec{v_2}) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\mathcal{B} = (\vec{v_1}, \vec{v_2})$

$T(\vec{v_1}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}^{=\vec{v_1}}$ in $\mathcal{B}$; $[T(\vec{v_1})]_{\mathcal{B}} = \begin{pmatrix} r \\ s \end{pmatrix}$ i.e. $T(\vec{v_1}) = r\vec{v_1} + s\vec{v_2}$

$r=1 \quad s=0$

$T(\vec{v_2}) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}^{=-\vec{v_2}}$ in $\mathcal{B}$? $[T(\vec{v_2})]_{\mathcal{B}} = \begin{pmatrix} r \\ s \end{pmatrix}$ i.e. $T(\vec{v_2}) = r\vec{v_1} + s\vec{v_2}$

$r=0 \quad s=-1$

$$\left[\begin{pmatrix}1\\2\\3\end{pmatrix}\right]_\beta$$

$$\left[\begin{pmatrix}1\\2\\3\end{pmatrix}\right]_\alpha = \begin{pmatrix}1\\2\\3\end{pmatrix}$$

$$\begin{pmatrix}1\\2\\3\end{pmatrix} = 1\vec{e_1} + 2\vec{e_2} + 3\vec{e_2}$$

$$[1+x+x^2]_\beta = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\parallel$$

$$[a+b(x-1)+c(x-1)^2]_\alpha = \begin{pmatrix}a\\b\\c\end{pmatrix}$$

$$[\nearrow]_\beta$$

$$\begin{pmatrix}1\\2\\3\end{pmatrix}$$

$\mathbb{R}^3$

$$S^{-1}AS$$

$$\beta \xleftarrow{} \alpha \xleftarrow{} \gamma \xleftarrow{} \ddot{\beta}$$

translate β $T(v_1)$

$$[T(v_1)]_\beta = \begin{pmatrix}r\\s\end{pmatrix}$$

T, $\vec{v_1}$ given

$$T(\vec{v_1}) = \vec{v_1}$$

$\begin{pmatrix}r\\s\end{pmatrix} \leftarrow$ solve for

$$\begin{pmatrix}1\\1\end{pmatrix} = r\vec{v_1} + s\vec{v_2}$$

$$\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}r+s\\r-s\end{pmatrix}$$

helpful
general

$r = 1$

$s = 0$

$$T(\vec{v_1}) = \begin{pmatrix}1\\1\end{pmatrix} = \vec{v_1}$$

$$T(\vec{v_2}) = \begin{pmatrix}-1\\1\end{pmatrix} = -\vec{v_2}$$

$$[T(\vec{v_1})]_\beta = \begin{pmatrix}1\\0\end{pmatrix}$$

$$[T(\vec{v_2})]_\beta = \begin{pmatrix}0\\-1\end{pmatrix}$$

$$A = \begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$B = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$B = \begin{pmatrix}[T(v_1)]_\beta & [T(v_2)]_\beta\end{pmatrix}$$

$$\begin{pmatrix}1\\0\end{pmatrix}\cancel{+\begin{pmatrix}0\\-1\end{pmatrix}}$$

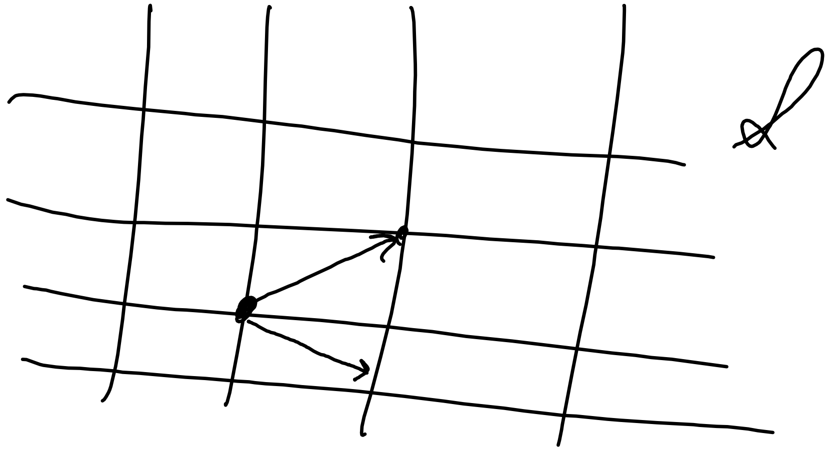
$$\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

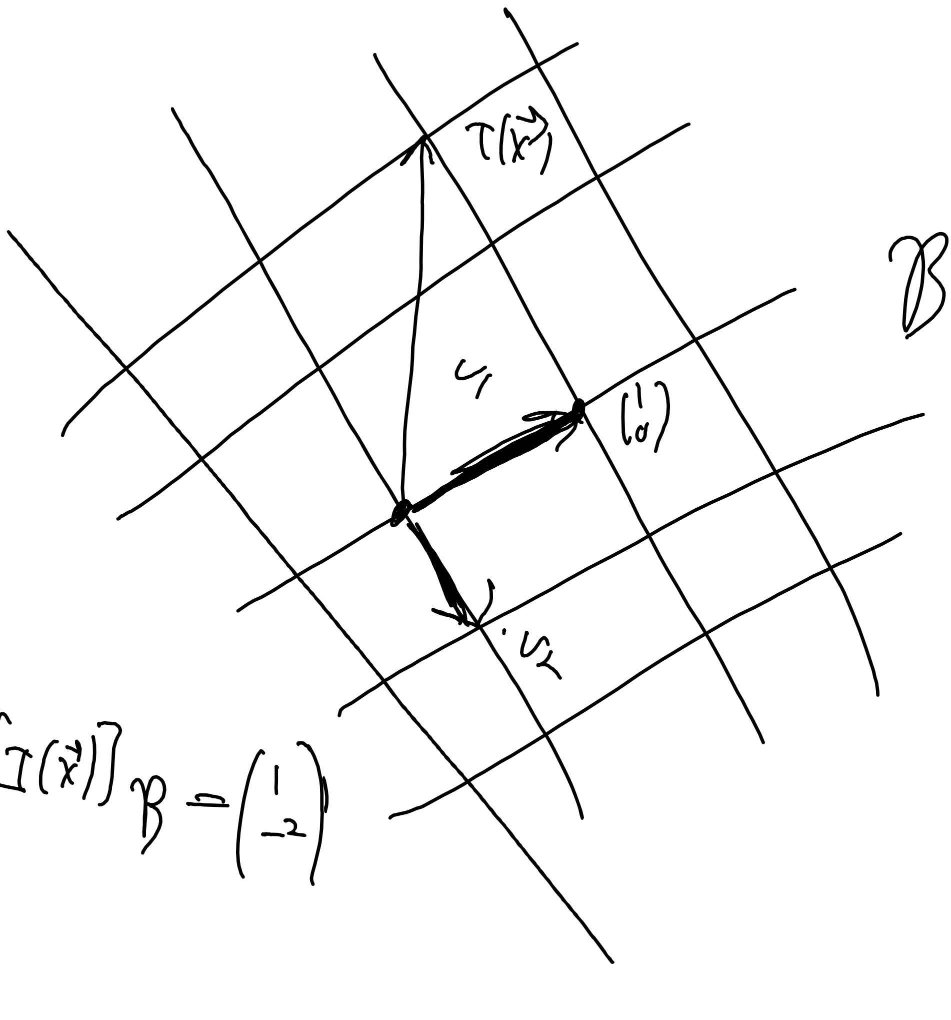
reflection

about $\begin{pmatrix}2\\3\end{pmatrix}$

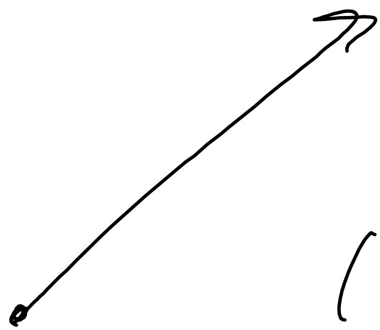
$$\begin{pmatrix}1&0\\0&-1\end{pmatrix}_\beta$$

& 3?





$$[T(\vec{x})]_{\mathcal{B}} = \begin{pmatrix}1\\-2\end{pmatrix}$$



$\left(t\vec{v}\right)$ $\begin{pmatrix} a \\ a \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ α

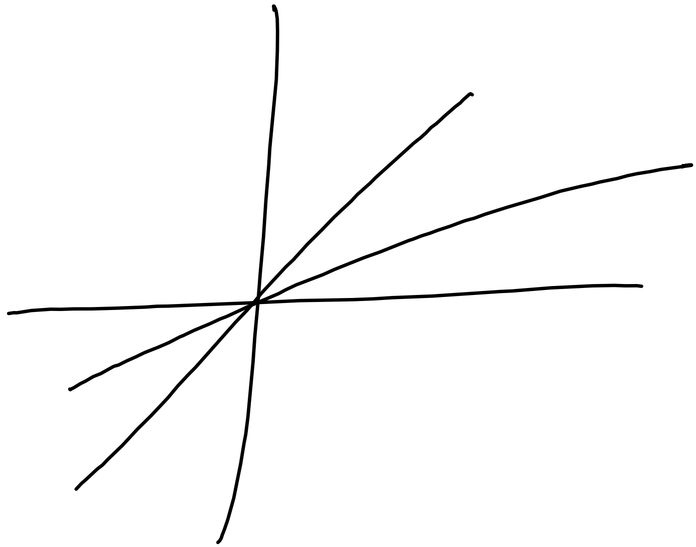
$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ β

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $(1/2, 1)$

$\begin{pmatrix} 7 \\ 2 \end{pmatrix}$,

$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$,

$\begin{pmatrix} 17 \\ 2, [illegible] \end{pmatrix}$,



$$\begin{pmatrix} \sqrt{3} \\ \pi \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$r\vec{v_1} + s\vec{v_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = T(\vec{v_1})$$

$$r'\vec{v_1} + s'\vec{v_2} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} = T(\vec{v_2})$$

$$B = \begin{pmatrix} r & r' \\ s & s' \end{pmatrix}$$

$$P = \begin{pmatrix} v_1 & v_2 \\ 1 & 1 \end{pmatrix}$$

$$P\begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$P\begin{pmatrix} r' \\ s' \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

change of basis

$$(P = S)$$ matrix

$$P[\vec{v_1}]_B = [v_1]_\alpha$$

$$P = {}_\alpha[I]_B$$