$\underset{2\times 2}{A}$, $\underset{2\times 2}{B}$

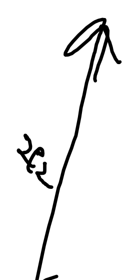
$\underset{2\times 2}{A}(B\,x)$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

90° rotation
?

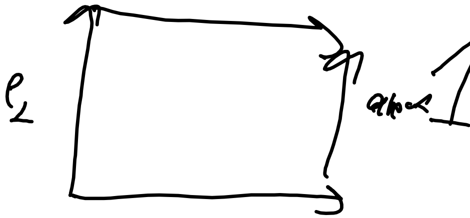
T, $\underline{T(\vec{e}_1)}$, $\underline{T(\vec{e}_2)}$

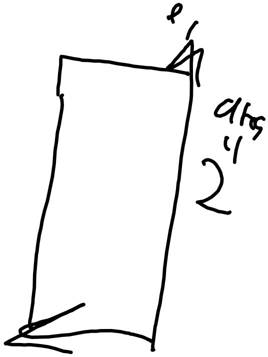
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} c \\ 1 \end{pmatrix}$



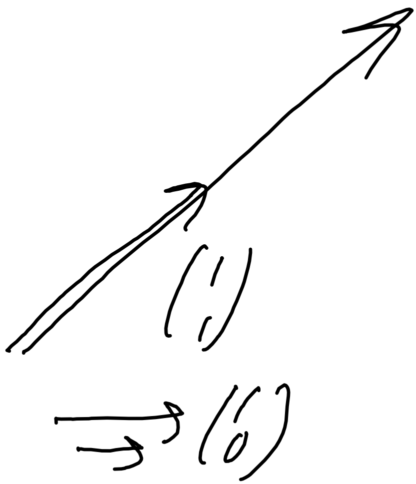
$$\begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ c & 2 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

2





twice the area



$\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ why is this rotation?

T is linear

$: \mathbb{R}^n \longrightarrow \mathbb{R}^m$

if there is some matrix $\underset{m \times n}{A}$

so that $T(\vec{x}) = A\vec{x}$

function eval — matrix product

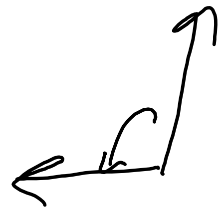
90° rotation ccwise

$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ $\begin{pmatrix}0\\1\end{pmatrix}$

$T(\vec{x}) = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\vec{x}$

$T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$ ✓

$$T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\0\end{pmatrix}$$



$\begin{pmatrix}-1\\0\end{pmatrix}$

$$\vec{e}_1=\begin{pmatrix}1\\0\end{pmatrix}, \quad \vec{e}_2=\begin{pmatrix}0\\1\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix} = x\begin{pmatrix}1\\0\end{pmatrix}+y\begin{pmatrix}0\\1\end{pmatrix} = x\vec{e}_1 + y\vec{e}_2$$

(Thm 2.1.3)

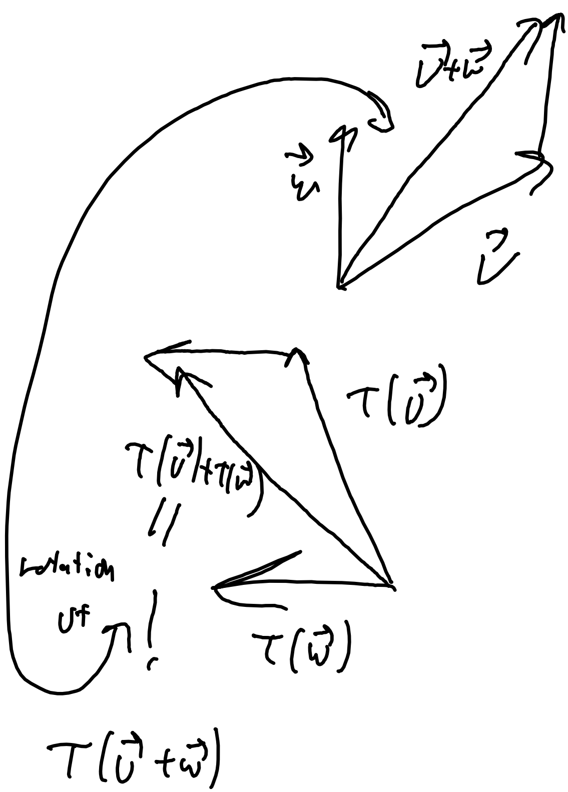
$$T(x\vec{e}_1 + y\vec{e}_2) = x\underbrace{T(\vec{e}_1)}_{\begin{pmatrix}0\\1\end{pmatrix}} + y\underbrace{T(\vec{e}_2)}_{\begin{pmatrix}-1\\0\end{pmatrix}}$$

$$= x\begin{pmatrix}0\\1\end{pmatrix}+y\begin{pmatrix}-1\\0\end{pmatrix} = \begin{pmatrix}-y\\x\end{pmatrix} = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

$$T(x\vec{e_1} + y\vec{e_2}) = x\,T(\vec{e_1}) + y\,T(\vec{e_2})$$

rotation

$$T(\vec{v} + \vec{w})$$



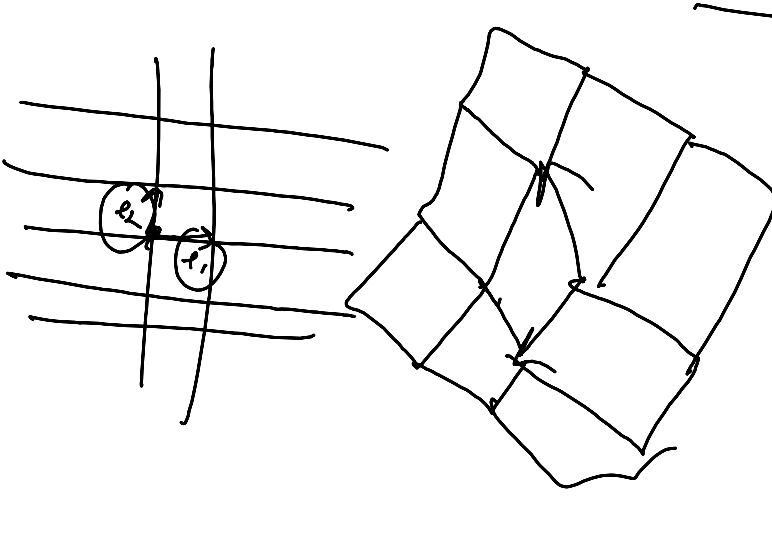
T, repr by $A = \begin{pmatrix} | & | & & | \\ \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \\ | & | & & | \end{pmatrix}$

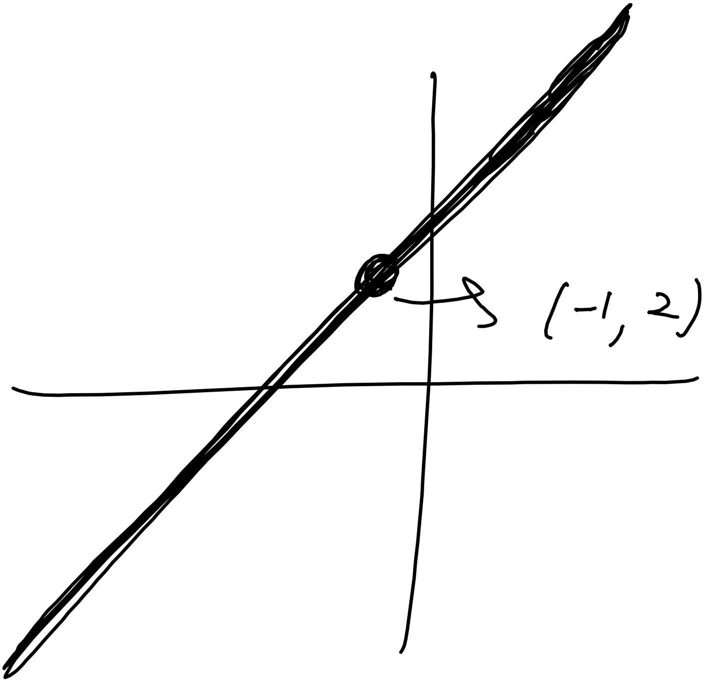
$\vec{v}_i = T(\vec{e}_i)$

$$T\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \underset{(?)}{=} x_1 \boxed{\vec{v}_1} + \cdots + x_n \boxed{\vec{v}_n}$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \underset{(?)}{=} x_1 \vec{e}_1 + \cdots + x_n \vec{e}_n$$

$$T(a\vec{v} + b\vec{w}) = aT(\vec{v}) + bT(\vec{w})$$





$(-1, 2)$

$T: \mathbb{R}^1 \longrightarrow \mathbb{R}^1$

1x1 matrix

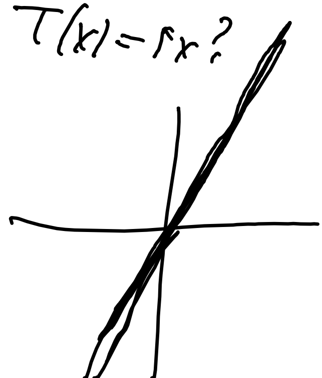
2x2 $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

1x1 (5)

$T(x) = Ax = 5x$

$[x]$

$T(x) = 5x$?

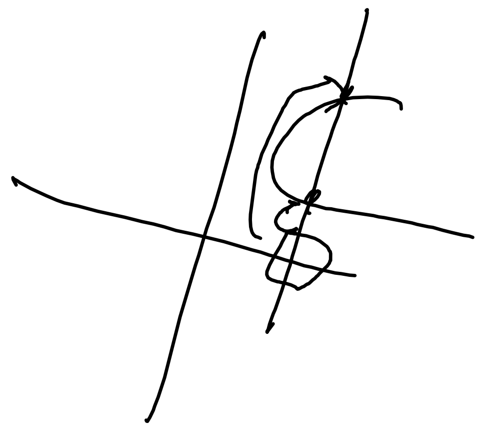


$$T(\vec{0}) = \vec{0}$$

$$A\begin{pmatrix}0\\0\\\vdots\\0\end{pmatrix} = \begin{pmatrix}0\\0\\\vdots\\0\end{pmatrix}$$

origin $\longmapsto$ origin

$(\underbrace{0}_{x}, \underbrace{0}_{y})$.



$$\boxed{T\begin{pmatrix} x \\ y \end{pmatrix} = x}$$

$$\left\{ \underbrace{\left(\vec{x}, T(\vec{x})\right)} \;\middle|\; \vec{x} \in \mathbb{R}^n \right\}$$

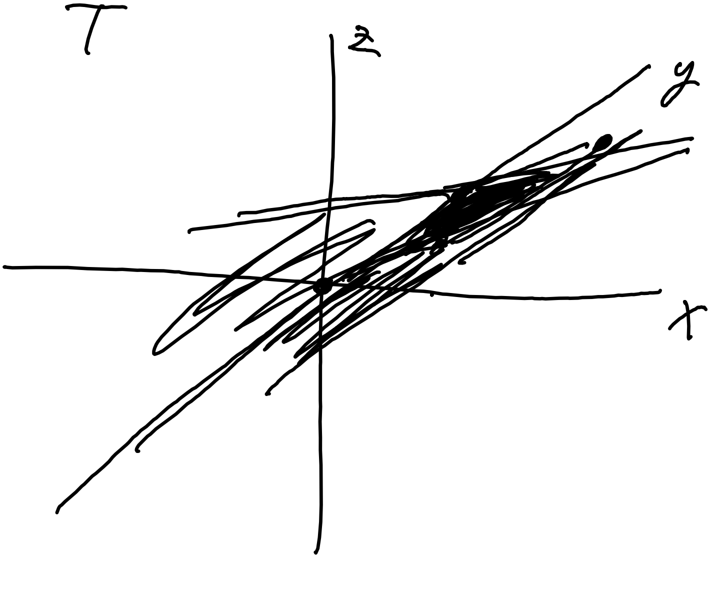
$T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$

$T(\vec{x})$ in $\mathbb{R}^m$

$\vec{x}$ in $\mathbb{R}^n$

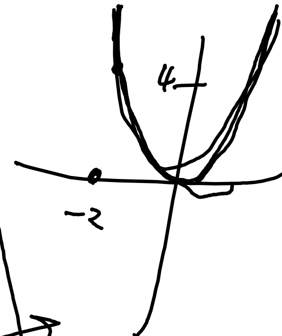
$(\vec{x}, \underline{T(\vec{x})})$ in $\underline{\mathbb{R}^{n+m}}$

T

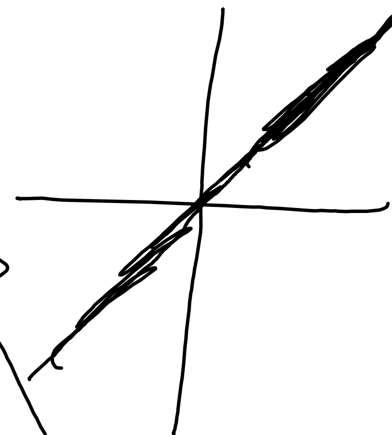


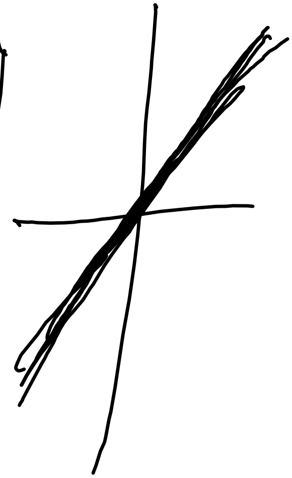
$T: \mathbb{R} \longrightarrow \mathbb{R}$ for $c \in \mathbb{R}$

$T(x) = cx$



$y = t(x) = x$





$$T\begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 0 \end{pmatrix}$$