$$\begin{cases} 2x + y + z = 1 \\ 3x + y = 2 \end{cases}$$

$$\begin{array}{ccc|c} x & y & z & = \\ \hline (2) & 1 & 1 & 1 \\ 3 & 1 & 0 & 2 \end{array}$$

$R_1 \longmapsto \frac{1}{2} R_1$

$$\left(\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ (3) & 1 & 0 & 2 \end{array}\right)$$

$$\begin{cases} x + \frac{1}{2}y + \frac{1}{2}z = 1/2 \\ 3x + y = 2 \end{cases}$$

$R_2 \longmapsto R_2 - 3R_1$

$$\left(\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & (-\frac{1}{2}) & -\frac{3}{2} & \frac{1}{2} \end{array}\right)$$

$R_2 \longmapsto -2$

$$\left(\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1/2 \\ 0 & 1 & 3 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1/2 \\ 0 & 1 & 3 & -1 \end{array}\right)$$

$R_1 \longmapsto R_1 - \frac{1}{2} R_2$

$\frac{1}{2} - \frac{1}{2}(-1)$

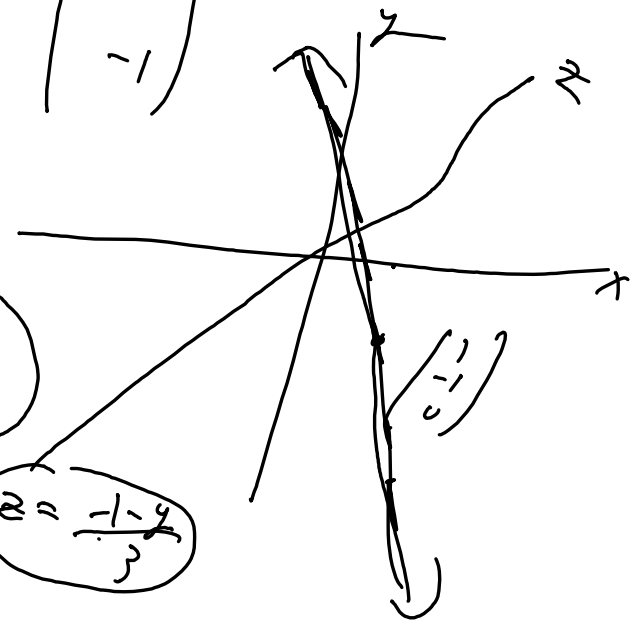
$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 3 & -1 \end{array}\right)$$

$z = x - 1$

$$\begin{cases} x - z = 1 \\ y + 3z = -1 \end{cases}$$

$$\begin{cases} x = 1 + z \\ y = -1 - 3z \end{cases}$$

$z = \frac{-1-y}{3}$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1+z \\ -1-3z \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}, \quad z \in \mathbb{R}$$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

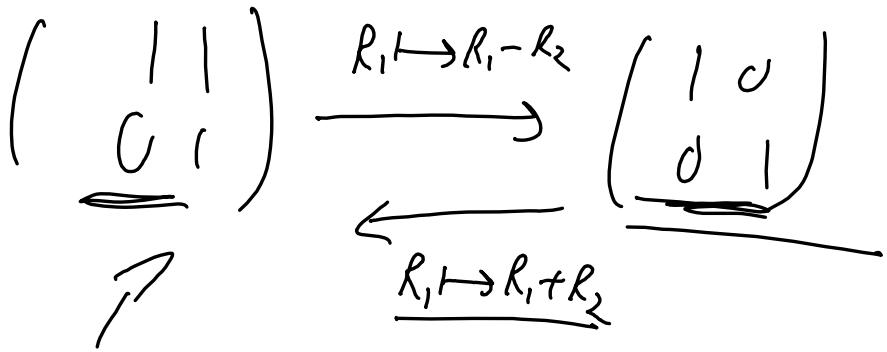
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & * \\ 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

1 1 2 3 4

$\begin{pmatrix} 1 & * & * & \cdots & * \end{pmatrix}$

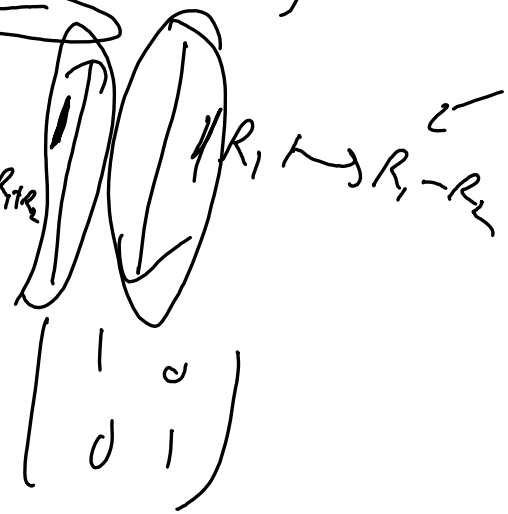
$\begin{pmatrix} 1 & 0 & * & \cdots & * \\ 0 & 1 & 0 & \cdots & 0 \end{pmatrix}$

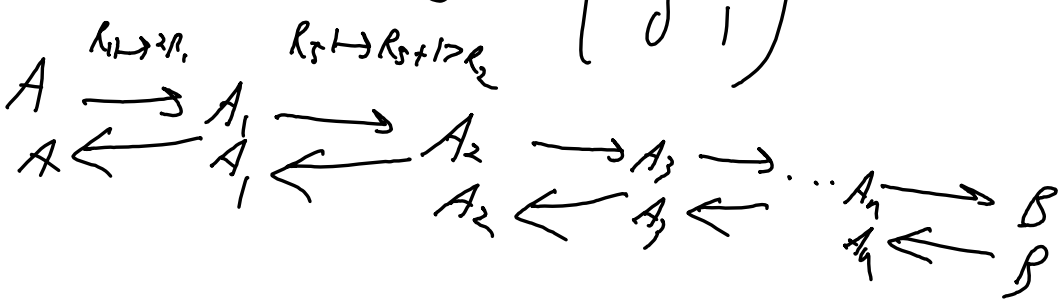
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$



$(AB)^{-1} = B^{-1} A^{-1}$

$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$





1, 2. 50

$$x_k = \frac{1}{2}(x_{k-1} + x_{k+1})$$

x # male children

y # female children

goal: $x+y$

5 male

5 female 4 brothers

Emile has $x-1$ brothers

11 has y sisters

$y=2(x-1)$, $y=2(y-2)$

$y=4$, $x=5$

girl has x brothers

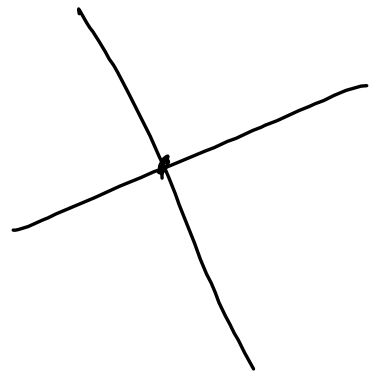
11 11 $y-1$ sisters

$x=y-1$

$y=2x-2$

$-2x+y=-2$

$x-y=-1$



$$\begin{pmatrix} 2 & 1 & -2 \\ 1 & -1 & -1 \end{pmatrix}$$

$R_2 \leftrightarrow R_1$

$$\begin{pmatrix} 1 & -1 & -1 \\ 2 & 1 & -2 \end{pmatrix}$$

$R_2 \mapsto -2R_2 + R_2$

$$\begin{pmatrix} 1 & -1 & -1 \\ 0 & -1 & -4 \end{pmatrix}$$

$R_1 \mapsto R_1 - R_2$

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & -1 & -4 \end{pmatrix}$$

$R_2 \mapsto -R_2$

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \end{pmatrix}$$

$x = 3$

$y = 4$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 6 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\mapsto$

$R_1 \mapsto R_1 - 2R_2$

$$\begin{pmatrix} 1 & 1 & 7 \\ 0 & -1 & -4 \end{pmatrix}$$

$x + y = 7$

$-y = -4$