Systems of linear equations

$1x + 1y + 1z = 1$

$2x + 3y + 4z = 2$

$x - y + z = 5$

$$\longrightarrow \begin{array}{c} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 2 \\ 1 & -1 & 1 & 5 \end{array}\right) \\ x \quad y \quad z \end{array}$$

row reduction
$\parallel$
Gauss – Jordan elimination

consistent? → 1 solution → what are they?
→ ∞ly many →
inconsistent?

1 equation — $ax = b$
1 unknown —

$x = \frac{b}{a}$ — $a \neq 0$ ✓

$a = 0$ → $b = 0$ ✓
→ $b \neq 0$ ✗ inconsistent

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$$

$$\overset{\parallel}{A} \qquad \overset{\parallel}{\vec{x}} \qquad \vec{b}$$

$$\boxed{A\vec{x} = \vec{b}}$$

$$A\vec{x} = \vec{b}$$

$$\vec{x} = A^{-1}\vec{b} \quad \text{(caveats)}$$

1 equation (multidimensional)

1 unknown (multidimensional) $\vec{x}$

subtleties! $\vec{b}$ can't always be reach

could be no solution

$\vec{x}$ is not always unique

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} x \\ 0 \end{pmatrix}$$

no solution

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right)$$

$0 = 1$ ✗

$$\boxed{\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

everything is a solution

$A\vec{x} = \vec{b}$ inconsistent iff rref(A) contains a row of zeros

false

$\vec{b} = \vec{0}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$\vec{x} = \vec{0}$

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 1 \\ 4 & 5 & 6 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \end{array}\right)$$

$x \; y \; z \; w$ → one variable is missing

- $x + 2y + 3z = 1$
- $4x + 5y + 6z = 2$
- $0 = 3$

eq w/ no variables → trivially true $0 = 0$

→ trivially false $0 = 1$, $0 = 3$

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 2 & 2 \end{array}\right) \overset{\text{swap row}}{\longleftrightarrow} \left(\begin{array}{cc|c} 2 & 2 & 3 \\ 1 & 1 & 1 \end{array}\right) \quad R_1/2$$

$R_2 - 2R_1$

$$\begin{pmatrix} x + y = 1 \\ 2x + 2y = 2 \end{pmatrix} \to \begin{pmatrix} x + y = 1 \\ 0 = 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 2x + 2y = 2 \\ 0 = 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & & 0\ \text{or}\ w \\ & \ddots & \\ 0\ \text{or}\ u & & 1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} (1\,0\,0)\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 1 \\ (0\,1\,0)\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 2 \\ (0\,0\,1)\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 3 \end{pmatrix}$$

$$I_n \vec{x} = \vec{b}$$

// unique solution $\underline{\vec{x} = \vec{b}}$

$\vec{x}$

A $m \times n$

$T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$

$\mathrm{im}(A) =$ span cols of A

$$A = \begin{pmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & & | \end{pmatrix}$$

$$A \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = x_1 \vec{v}_1 + \cdots + x_n \vec{v}_n$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

basis?

ker(A)?

$A\vec{x}=\vec{0}$

Gauss Jordan elimination

$$\text{im}(A) = \text{span}\left\{ \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 6 \end{pmatrix} \right\}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ -4 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1/3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

$\vec{a}$ $\vec{b}$ $\vec{c}$

$$\vec{c} = -\vec{a} + 2\vec{b}$$

$$\begin{pmatrix} -1 \\ 2 \end{pmatrix} = -1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 6 \end{pmatrix} = -1\begin{pmatrix} 1 \\ 4 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

basis im(A)

rref(A), find cols w/ pivots = leading 1, those corresponding columns are the basis

algorithm

$$\begin{pmatrix} 2 & \\ & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & a \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ c & d \end{pmatrix}$$

multiply 1st row of A by 2

$R_1 \longmapsto 3R_2 + R_1$

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+3c & b+3d \\ c & d \end{pmatrix}$$

<u>rref(A)</u>

$$\underline{E_n \cdots E_1 E_2 E_1 A}$$

$\mathrm{rref}(A) = EA$ E invertible matrix

same kernel

6. Let $\vec{v}$ be in $\mathbb{R}^3$

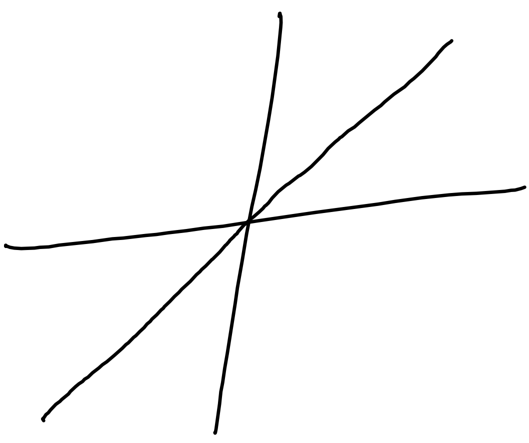
$$T(\vec{w}) = \vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

$\ker(T)$? $\operatorname{im}(T)$?

geometrically and algebraically

describe the picture

"formula"
spanning set
basis.



xy-plane in $\mathbb{R}^3$

basis $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}$

$\operatorname{span}\left(\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right)$

$\left\{ \begin{pmatrix}x\\y\\0\end{pmatrix} \right\}$ x, y in $\mathbb{R}$

"formula"

$T(\vec{e_1})$ $\quad T(\vec{e_2})$ $\quad T(\vec{e_3})$

$\vec{v} \cdot \vec{e_1}$

v_1 $\quad v_2$ $\quad v_3$

matrix	"abstract reasoning"
"mechanical"	rank-nullity thm

$(v_1 \quad v_2 \quad v_3)$

$\vec{v} = 0$

$\vec{v} \neq 0$

$\mathbb{R}^3 \xrightarrow{T} \mathbb{R}^1$

dimension counting

$3 = \text{null}(T) + \underbrace{\text{rank}(T)}_{0,\ 1}$

$\text{rank}(T) = 0$

$\text{null}(T) = 3$

$\{0\} = \text{im}(T)$

$\ker(T) = \mathbb{R}^3$

$\vec{e}_1, \vec{e}_2, \vec{e}_3$

$\text{rank}(T) = 1 \iff \vec{v} \neq 0$

$T(\vec{v}) = \|\vec{v}\|^2 \neq 0$

ker

im

$\text{rank}(T) = 1$, $\text{im}(T)$ is a line

$\text{im}(T)$ contained in $\mathbb{R}$

---

1 d subspace of $\mathbb{R}$ must be all of $\mathbb{R}$

↳ V subspace $\mathbb{R}^m$

$\dim V = k$

$\underline{W}$ contained in $\underline{V}$ $\quad \dim W = \dim V$

V contained in W $\quad \underline{W = V}$

$\text{im}(T) = \mathbb{R}$

$\{1\}$ $\quad \{2\}$ $\quad \{-1\}$ $\quad \{\pi\}$

$\{\tfrac{1}{2}\}$

$\ker(T)$? $\quad \text{null}(T) = \dim \ker(T)$

$\quad \| $

$\quad 2$

$\ker(T) \subseteq \mathbb{R}^3$ $\quad \underline{\text{plane}}$

contained in

what plane is this?

ker(T)

$\vec{w} \mid T(\vec{w})=0$

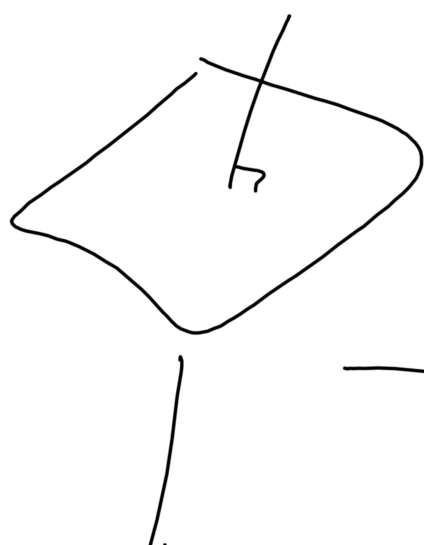
$\downarrow$

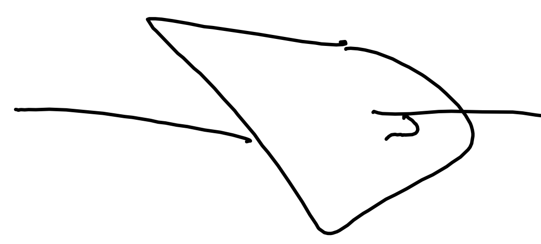
$\vec{v}\cdot\vec{w}$

$\vec{w}$ s.t. $\vec{v}\cdot\vec{w}=0$

$\vec{v}\perp\vec{w}$

the plane perpendicular to $\vec{v}$ geometric

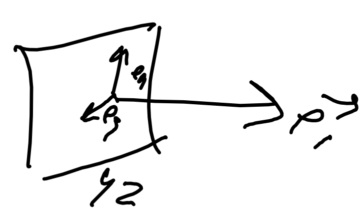




$\vec{v}=\vec{e_1}$, what is ker(T)

yz-plane

basis (e_2, e_3)



basis ker(T)?

$\vec{x}, \vec{y}$

$\vec{v} = \begin{pmatrix}1\\2\\3\end{pmatrix}$

$T(\vec{w}) = \begin{pmatrix}1\\2\\3\end{pmatrix} \cdot \vec{w}$

$T(\vec{w}) = w_1 + 2w_2 + 3w_3$

$T(\vec{w}) = 0$

$w_1 + 2w_2 + 3w_3 = 0$

$(1 \quad 2 \quad 3 \mid 0)$

$w_1 = -2w_2 - 3w_3$

$\boxed{w_3 = 0}$

$w_1 = -2w_2$

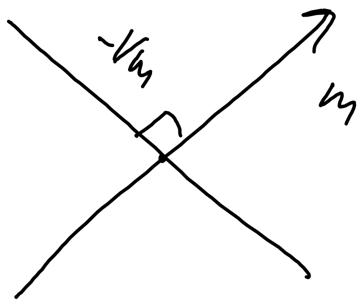
$\to w_2 = 0 \implies w_1 = 0$ $\begin{pmatrix}0\\0\\0\end{pmatrix}$ in ker(T)

$\to w_2 = 1 \quad w_1 = -2$ $\begin{pmatrix}-2\\1\\0\end{pmatrix}$

$$\begin{pmatrix}1\\2\\3\end{pmatrix} , \begin{pmatrix}-2\\1\\0\end{pmatrix} = 0$$

$$\begin{pmatrix}1\\2\end{pmatrix} , \begin{pmatrix}-2\\1\end{pmatrix}$$

$$\begin{pmatrix}-2\\1\\0\end{pmatrix}$$



lin indep

$\begin{pmatrix}-2\\1\end{pmatrix}$ $\ker(T)$

$\cancel{w_3 = 0}$

$w_1 = 0$. $2w_2 + 3w_3 = 0$

$w_3 = 1$, $2w_2 + 3 = 0$

$w_2 = -\frac{3}{2}$

$$\begin{pmatrix}0\\-3/2\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\-3/2\\1\end{pmatrix} , \begin{pmatrix}1\\2\\3\end{pmatrix} = 0$$

$\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -3/2 \\ 1 \end{pmatrix}$ in Ker(T)

dim Ker T = 2

basis!

general?

$\vec{v} \neq 0$

solve this for $\vec{w}$

$\Leftrightarrow v_1 w_1 + v_2 w_2 + v_3 w_3 = 0$

$w_3 = 0$?

$v_1 w_1 + v_2 w_2 = 0$

$w_2 = 1$

$v_1 w_1 + v_2 = 0$

$w_1 = \frac{-v_2}{v_1}$

$\vec{w} = \begin{pmatrix} -v_2/v_1 \\ 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -v_2/v_1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0$

Case 1. $v_1 \neq 0$

as before $\vec{v} = \begin{pmatrix} -v_2/v_1 \\ 1 \\ 0 \end{pmatrix}$

$w_2 = 0$

$v_1 w_1 + v_3 w_3 = 0$

$w_3 = 1$

$v_1 w_1 + v_3 = 0$

$w_1 = \dfrac{-v_3}{v_1}$

$\begin{pmatrix} -v_3/v_1 \\ 0 \\ 1 \end{pmatrix}$

lin indep

Case 2. $v_1 = 0$

$w_2 v_2 + w_3 v_3 = 0$

$\begin{pmatrix} 0 \\ v_2 \\ v_3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -v_3 \\ v_2 \end{pmatrix} = 0$

$v_2 = 0$

$w_3 v_3 = 0$

w_2 free

$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$v_2 \neq 0$, $w_3 = 1$

~~$w_3 = 0$, $w_2 v_2 = 0$~~ $\Rightarrow w_2 = 0$

$w_2 v_2 + w_3 = 0$

$w_2 = \dfrac{-v_3}{v_2} \rightarrow \begin{pmatrix} 1 \\ -v_3/v_2 \end{pmatrix} \; // \; \begin{pmatrix} 0 \\ v_2 \\ -v_3 \end{pmatrix}$

4. reflection $\mathbb{R}^3 \longrightarrow \mathbb{R}^3$

about plane $y = z$

find the matrix for the linear transformation $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

given by reflection about plane $y = z$

what is the matrix for T?

3×3 matrix, ~~A~~

$\mathbb{R}^3 \to \mathbb{R}^3$

$\mathbb{R}^n \longrightarrow \mathbb{R}^m$

$m \times n$

first col $T(\vec{e_1})$

second $T(\vec{e_2})$

third $T(\vec{e_3})$

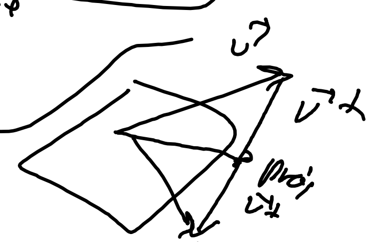
$T(\vec{x}) = A\vec{x}$

formula

Strategy: find $T(\vec{e_i})$ $i=1,2,3$.

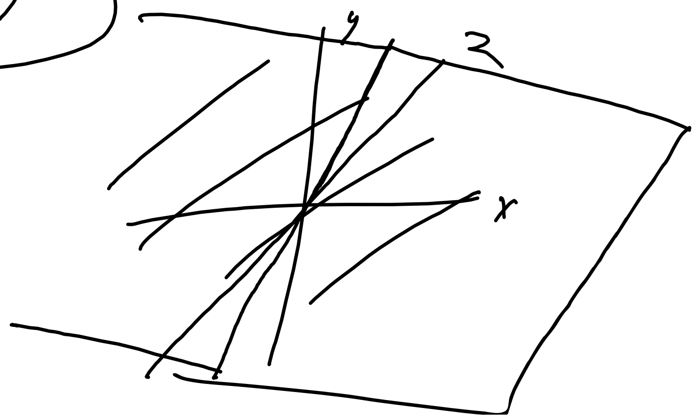
another approach: remember formula

proj onto plane



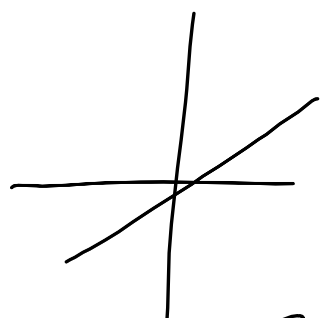
$\vec{v} - 2\vec{v}^{\perp} =$ reflected

(x, y, z)



$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$ normal vector

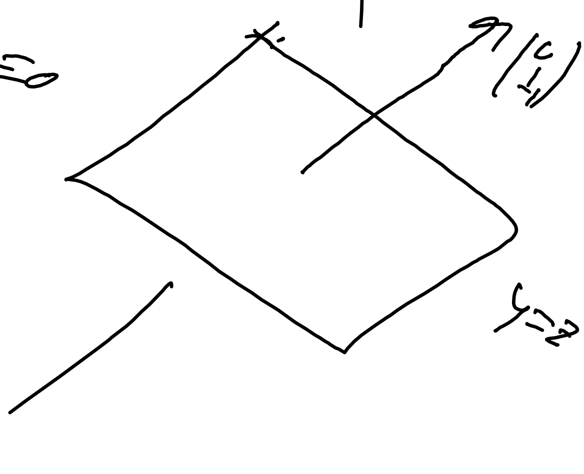
$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$



$y = z$

$y - z = 0$

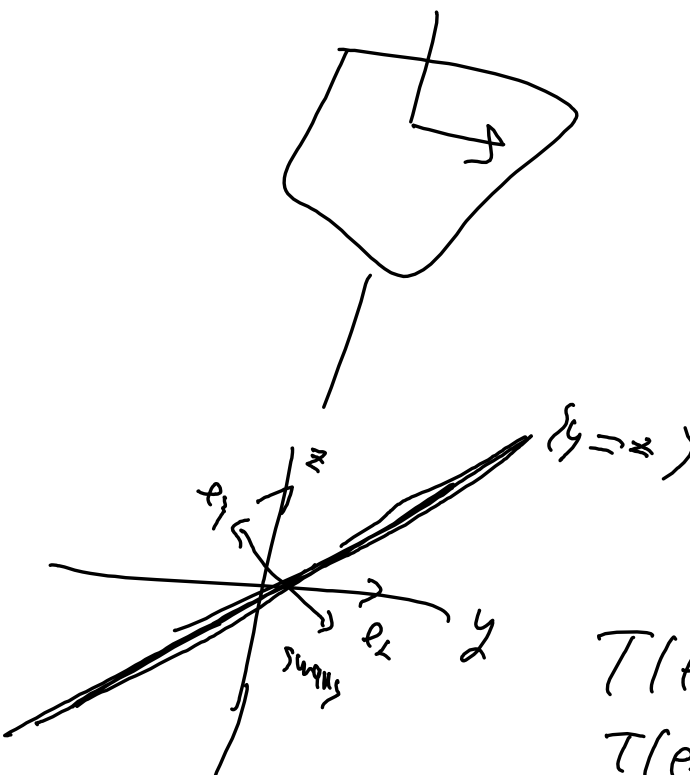
$0x + y - z = 0$



x-axis contained $\{y = z\}$

$$\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$$

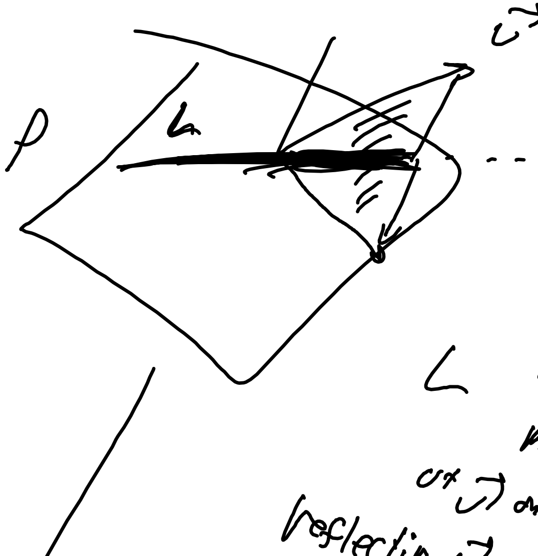
$\vec{e_1}$ in plane, $T(\vec{e_1}) = \vec{e_1}$



Cross section

$T(\vec{e_2}) = \vec{e_3}$

$T(\vec{e_3}) = \vec{e_2}$



L "shadow"
proj
$\vec{u}$ onto P

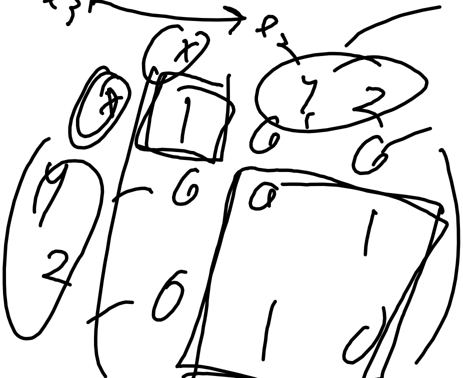
reflection $\vec{u}$ about P

reflection of $\vec{u}$ about L

$e_1 \longmapsto e_1$

$e_2 \longmapsto e_2$

$e_3 \longmapsto e_3$



"block diagonal"

$\sqrt{2}x + \pi y + e z = 0$

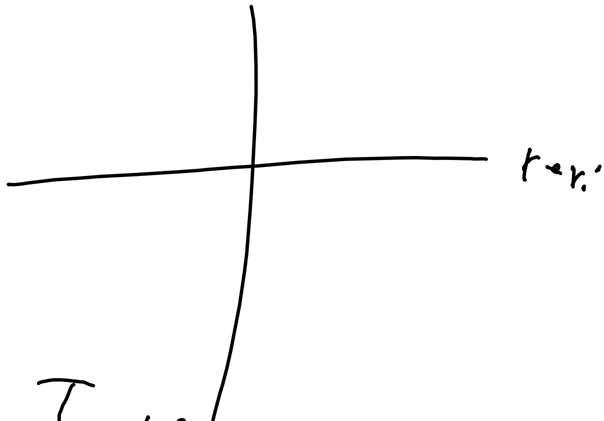
$\vec{e_1} \mapsto ??$

$$\begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

$-e_1$

$-e_2$

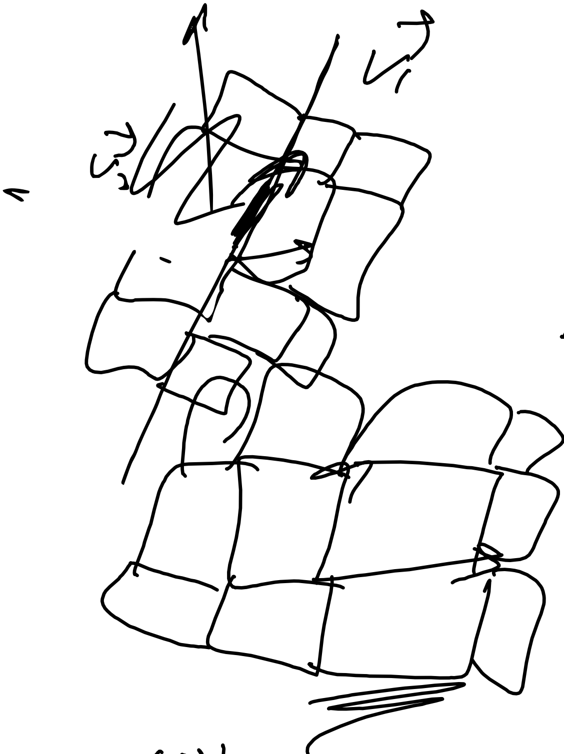
$-e_3$



T refl about x-axis

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



$T(\vec{v_1}) = \vec{v_1}$ reflections about $\vec{v_1}$

$T(\vec{v_2}) = -\vec{v_2}$

in basis $\mathcal{B} = (\vec{v_1}, \vec{v_2})$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{\mathcal{B}}$$

$$\begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}_{\mathcal{B}}$$

d)

A comm B

B comm C

A com C

A "commutes" w/ B

if $AB = BA$

~~AB^2CBA~~

A $n \times m$

B $k \times l$

$A \cdot B$

$n = k$

$n \times l$

$(n \times n)(m \times l) = n \times l$ $n \times l$

BA $k \times m$

$n = l$

$\underset{n \times l}{AB} = \underset{k \times m}{BA}$

$n = n$ $m = l$

A, B, C square matrices

same dimension

$AB = BA$

$BC = CB$

] hypotheses

$\Downarrow$

$AC = CA$] goal

A rot

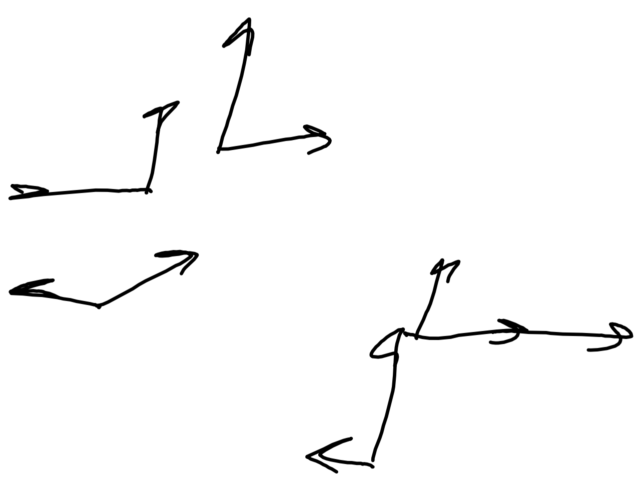
B scale $\longleftrightarrow$!!

C shear (?)

fact: A commutes w/ every matrix B

A scale

$A = cI$



Say true

$B = I$, or any scalar

$AB = BA$ true for all A

$CB = BC$ true for all C

$\Big\Downarrow$ "given true"

A commutes w/ C

\begin{tikzcd}
\underline{\mathcal{B}} \arrow[rr] \arrow[dr] & & \underline{\mathcal{R}} \\
& \underline{\mathcal{I}} \arrow[ur] &
\end{tikzcd}

$$\rho\,[x]_{\mathcal{B}} = [x]_{\mathcal{R}}$$

$$\underline{{}_{\mathcal{R}}[I]_{\mathcal{B}}}\ \underline{[x]_{\mathcal{B}} = [x]_{\mathcal{R}}}$$

$\rho\,\vec{e_i}$ i^{th} col of ρ

$$\begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \end{pmatrix}$$

$\underline{\rho\,\vec{e_i}}$ $\vec{e_i} = \underline{[\vec{x}]_{\mathcal{B}}}$

$$\underline{\rho\,[\vec{x}]_{\mathcal{B}} = [\vec{x}]_{\mathcal{R}}}$$

$$[\vec{x}]_\beta = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\vec{x} = 1\vec{v}_1 + 0\vec{v}_2 = \vec{v}_1$$

$$\beta = (\vec{v}_1, \vec{v}_2)$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$[\vec{v}_1]_\beta = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$P[\vec{v}_1]_\beta = [\vec{v}_1]_\gamma$$

$$\parallel$$

$$P\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

first col of P

$$P[\vec{v}_2]_\beta = [\vec{v}_2]_\gamma$$

$$\parallel$$

$$P\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2nd col

$$P = \begin{pmatrix} | & | \\ [v_1]_\gamma & [v_2]_\gamma \\ | & | \end{pmatrix}$$

$[\vec{v}_i]_R = \begin{pmatrix} a \\ b \end{pmatrix}$ $\quad R = (\vec{u}_1, \vec{u}_2)$

$\vec{v}_i = a\vec{u}_1 + b\vec{u}_2$

solve a, b

$$\begin{pmatrix} | & | \\ u_1 & u_2 \\ | & | \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \vec{v}_i$$

$$\begin{pmatrix} a_i \\ b_i \end{pmatrix} = \begin{pmatrix} \vec{u}_1 & \vec{u}_2 \\ | & | \end{pmatrix}^{-1} \vec{v}_i$$

$$\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} = P$$

P

${}_R P_B \quad {}_R[\ ]_B$

${}_S P_R \quad ({}_S P_B)^{-1} \quad {}_B P_S$

$$({}_B P_S \; {}_S P_R)^{-1} = ({}_B P_R)^{-1}$$

$$\beta = (\vec{v_1}, \ldots, \vec{v_n}) \qquad \mathcal{S}$$

$$P = \begin{pmatrix} | & & | \\ \vec{v_1} & \cdots & \vec{v_n} \\ | & & | \end{pmatrix}$$

$$P\vec{e_i} = \vec{v_i}$$

$$\|$$

$$P\left(\boxed{[v_i]_\beta}\right) = \vec{v_i} = [\vec{v_i}]_{\mathcal{S}}$$

e_i unitari

β can

$$[e_i]_\beta \neq \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}_{\mathcal{S}}$$

$$_{\mathcal{S}}P_\beta \, [v_i]_\beta = [\vec{v_i}]_{\mathcal{S}}$$

$$\begin{pmatrix} 0 & 1 & 2 & 1 & a \\ 0 & 0 & 0 & 1 & b \\ 3 & 4 & 5 & 1 & c \\ 0 & 0 & 6 & 1 & d \end{pmatrix}$$

1 ? eliminate

rk = 3

im(?) ≤ $\begin{pmatrix} a \\ a \\ c \\ d \end{pmatrix}$

4x4

b=0

if rk ≤ 3

rk=4

$$\begin{pmatrix} 0 & 1 & 2 & a \\ 0 & 0 & 0 & b \\ 3 & 4 & 5 & c \\ 0 & 0 & 6 & d \end{pmatrix}$$

rk=4

b≠0

$$\begin{pmatrix} 0 & 1 & 2 & 1 & a \\ 0 & 0 & 0 & 1 & b \\ 3 & 4 & 5 & 1 & c \\ 0 & 0 & 6 & 1 & d \end{pmatrix}$$

b=a

2b=0

$$\begin{pmatrix} 0 & 1 & 2 & 1 & a \\ 3 & 4 & 5 & 1 & c \\ 0 & 0 & 6 & 1 & d \end{pmatrix}$$

a, c, d for solvable

$$\sim \begin{pmatrix} 3 & 4 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 6 \end{pmatrix} \quad \det = 18$$

$b=0$

$\dim \operatorname{im}(4 \times 3 \text{ I wrote}) \Rightarrow 3$

$V = \left\{ \begin{pmatrix} a \\ 0 \\ c \\ d \end{pmatrix} \text{ in } \mathbb{R}^4 \right\}$ $\dim V = 3$

im(that guy) contain in V

$\begin{pmatrix} a \\ c \\ c \\ d \end{pmatrix}$