Proofs

Communication

logical + rigorous

↳ satisfy pedants

Examples

for all x ______ false

1. Everyone in this Zoom call is named "Jas Singh".

false, someone is named "Alondra Rosas" ≠ "Jas Singh"

for all x ______ True

2. Everyone in this Zoom call is in an upper division math class.

True, Let x be a person in the Zoom call. Then x is in 115A. 115A is an upper div math class, so x is in an upper div math class.

Thus, all people in the Zoom call are in an upper div math class.

There is x ______ True

3. Someone in this zoom call is named "Jas Singh".

True. The person sharing screen is named Jas.

4. Someone in this zoom call is a dragon.

false, check every person

There is x ______ false

Fields

$$\mathbb{F}_2 = \{\overset{a}{0}, \overset{b}{1}\}$$

$0 + \underbrace{y}_{} = 0$

+	0	1
0	0	1
1	1	0

(xor)

·	0	1
0	0	0
1	0	1

(and)

Prop. $(\mathbb{F}_2, +, \cdot)$ is a field.

Pf. We will check that the field are satisfied.

Commutativity. Claim. For all x, y in $\mathbb{F}_2$, $x+y = y+x$

$0+0 = 0 = 0+0$

$0+1 = 1 = 1+0$

$1+0 = 1 = 0+1$

$1+1 = 0 = 1+1$

for all x, y in $\mathbb{F}_2$, $x \cdot y = y \cdot x$

$0 \cdot 0 = 0$

$0 \cdot 0 = 0$

Thus, $0 \cdot 0 = 0 \cdot 0$

$0 \cdot 1 = 0$

$1 \cdot 0 = 0$

Thus, $0 \cdot 1 = 1 \cdot 0$

$1 \cdot 1$

Associativity.

for all x, y, z in $\mathbb{F}_2$ we have $(x+y)+z = x+(y+z)$

$x=1,\ y=0,\ z=1$

$(x+y)+z = (1+0)+1 = 1+1 = 0$

also $x+(y+z) = 1+(0+1) = 1+1 = 0$

so $(1+0)+1 = 1+(0+1)$

8 cases

Additive identity. Claim. 0 is an additive identity, i.e.

for all $x \in \mathbb{F}_2$, $x+0 = x$.

$x=0$, $0+0=0$ by def.

$x=1$, $1+0=1$ by def.

Thus, 0 is an additive identity

Multiplicative identity. Claim 1 is a multiplicative identity

That is, $x \cdot 1 = x$ for all x in $\mathbb{F}_2$

$x=0$, $0 \cdot 1 = 0$ by def

$x=1$, $1 \cdot 1 = 1$ by def

Thus, 1 is a multiplicative identity

Additive inverse. (for all x in $\mathbb{F}_2$ (there is y in $\mathbb{F}_2$ (so that $x+y=0$)))

depends on x

$x=0$. Want to show that there is y in $\mathbb{F}_2$ so that $0+y=0$.

Claim $y=0$ works

Indeed, $0+0=0$ by def.

$x=1$. Want to show that there is y in $\mathbb{F}_2$ so that $1+y=0$.

Claim $y=1$ works.

Indeed, $1+1=0$ by def

Multiplicative inverse.

$\{1\}$

For all (x in $\mathbb{F}_2$ so that $x \neq 0$), there is some y in $\mathbb{F}_2$ so that $x \cdot y = 1$

~~$x = 0$.~~

$x = 1$. Claim is that there is some y in $\mathbb{F}_2$ so that $1 \cdot y = 1$.

Claim $y = 1$ works.

Indeed, $1 \cdot 1 = 1$ by def.

Thus, multiplicative inverses exist for all nonzero elements of $\mathbb{F}_2$

For all x, y, z in $\mathbb{F}_2$, show

Distributivity: $x(y+z) = xy + xz$

$$\begin{array}{ccc} 0 & 0 & 0 \\ 1 & 1 & 1 \end{array}$$

$1 \cdot (0+1) = 1 \cdot 1 = 1$

or, $1 \cdot 0 + 1 \cdot 1 = 0 + 1 = 1$

Thus, $1 \cdot (0+1) = 1 \cdot 0 + 1 \cdot 1$

1, 2, 15

$V = \mathbb{R}^n$ ← vs / $\mathbb{R}$

vs / $\mathbb{C}$

← cordinate in $\mathbb{R}$

$$\lambda \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \in \mathbb{R}^n$$

$$\vec{v} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

$a_1 + b_1$

$a_1 b_1$

$$\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} + \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ \vdots \\ a_n + b_n \end{pmatrix}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_2 \\ a_2 + b_1 \end{pmatrix}$$

$0 \in V$ dem on $+$

$x + 0 = x$

$\mathbb{R}^2$

$\vec{0} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ← ausfuhks

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \oplus \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 - 1 \\ y_1 - 1 \end{pmatrix} + \begin{pmatrix} x_2 - 1 \\ y_2 - 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$