OH

Today 12PM

next Tu - 12

go over midterm

evals due next Tu 9am

Spanning trees

G connected $\boxed{\text{graph}}$,

assume today undirected

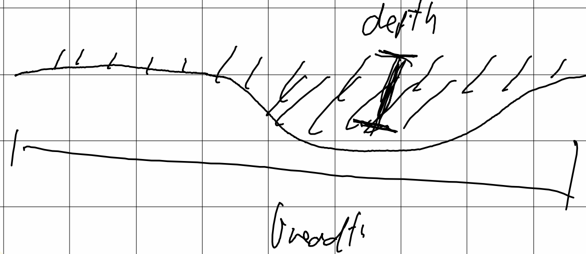
T subgraph of G

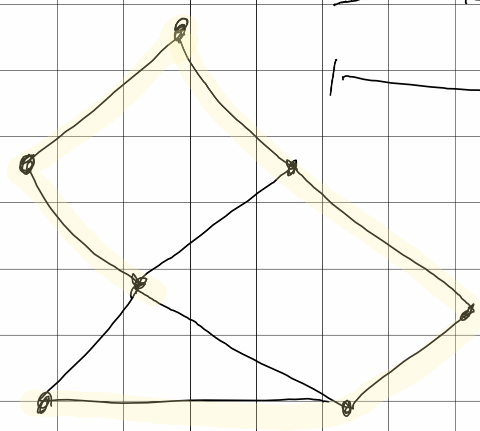
$(T \subseteq G)$

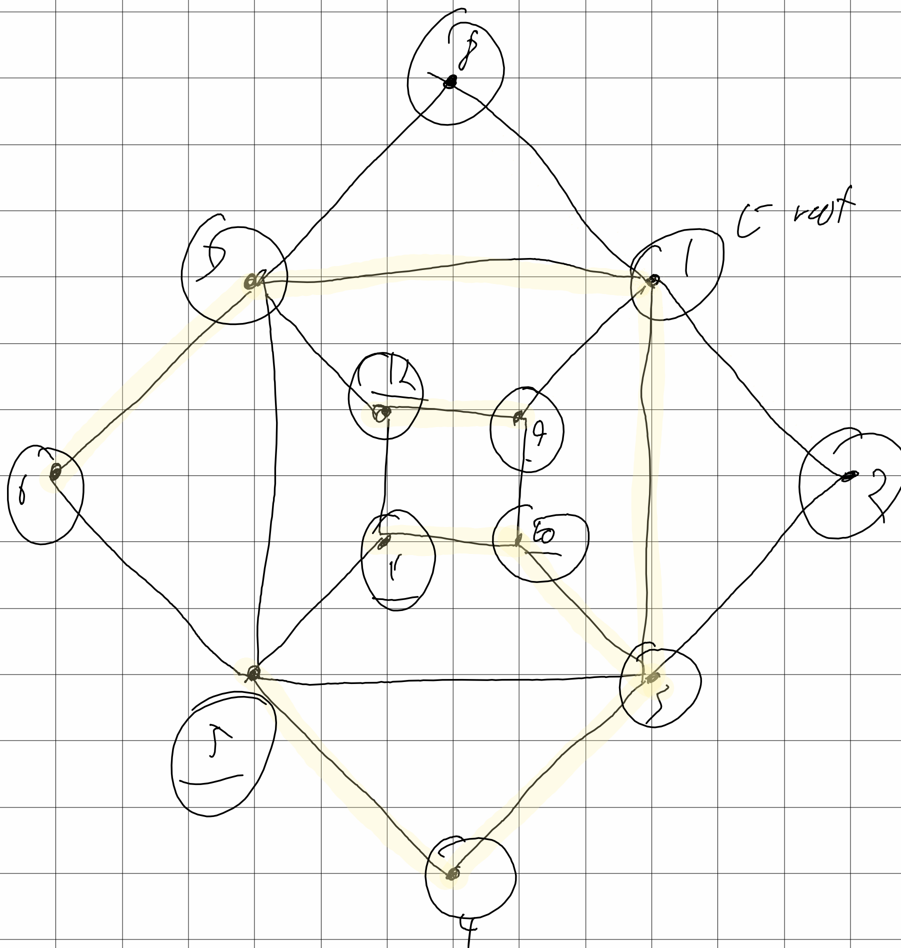
T is a spanning tree of G

if

1. T tree
2. $V(T) = V(G)$







Breadth first

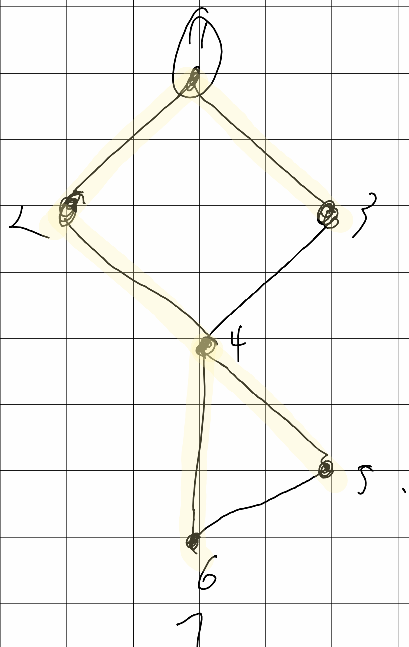
build T iteratively

begin w/ root

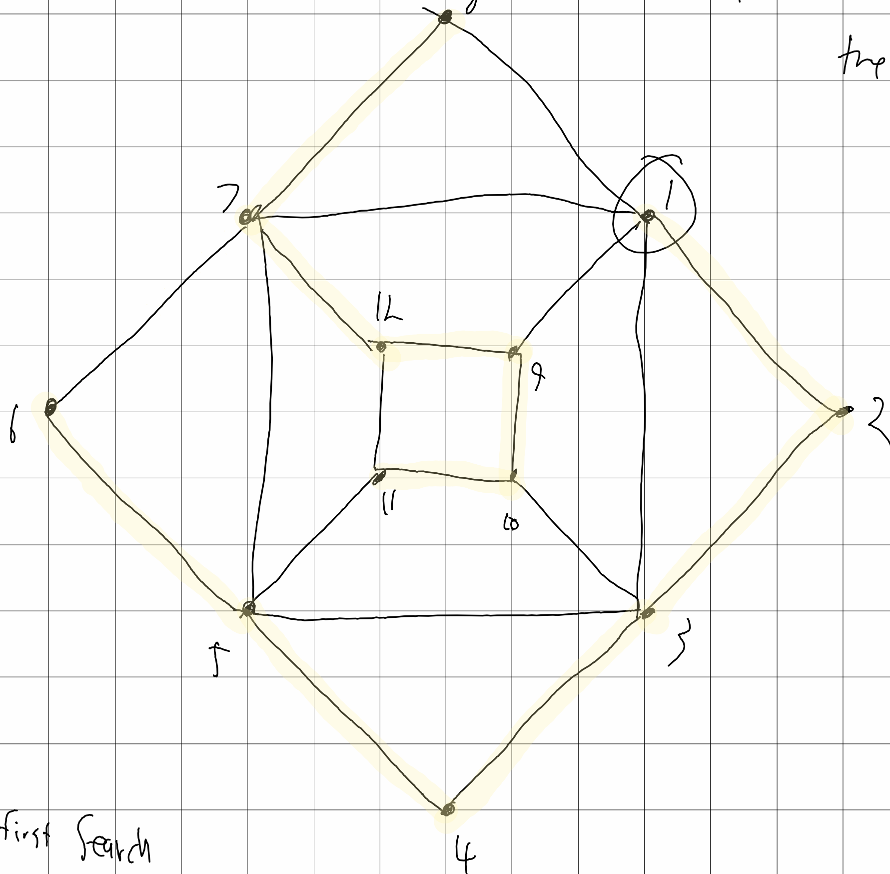
frontier = adjacent to root

add all these

frontier = adjacent to old frontier


1
2
3
4
5
6
7

no edges not touching the tree



depth first search

DFS

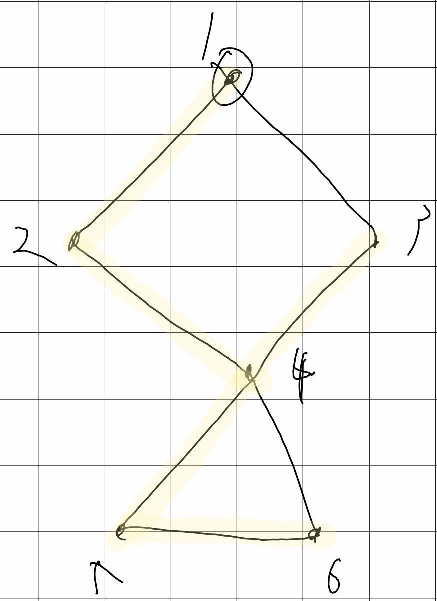
Start @ root . one vertex not on tree

add edges connected to current vertex w/ min'l vertex

this new one becomes current vertex

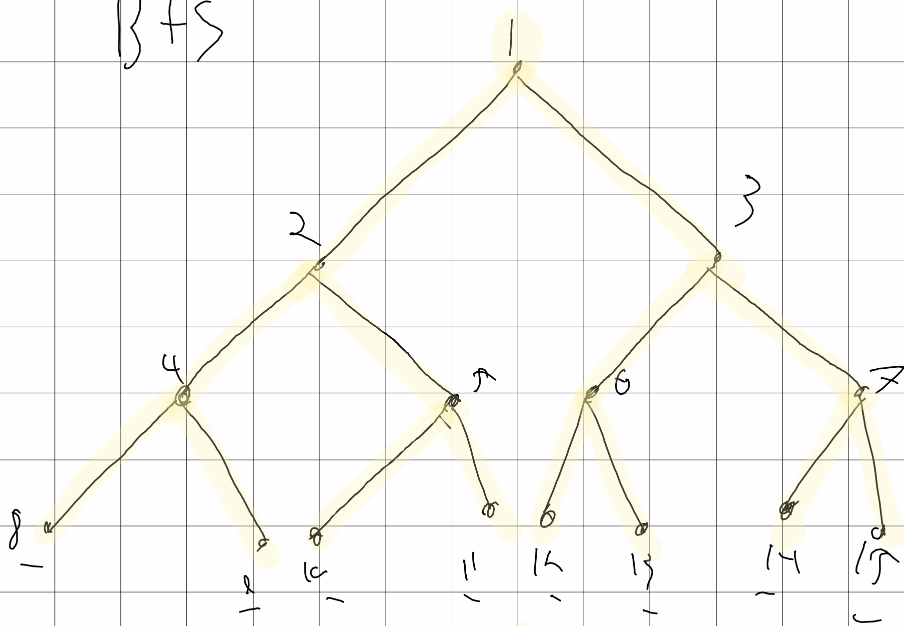
repeat until impossible

once stopped, backtrack and repeat


1
2
3
4
7
6

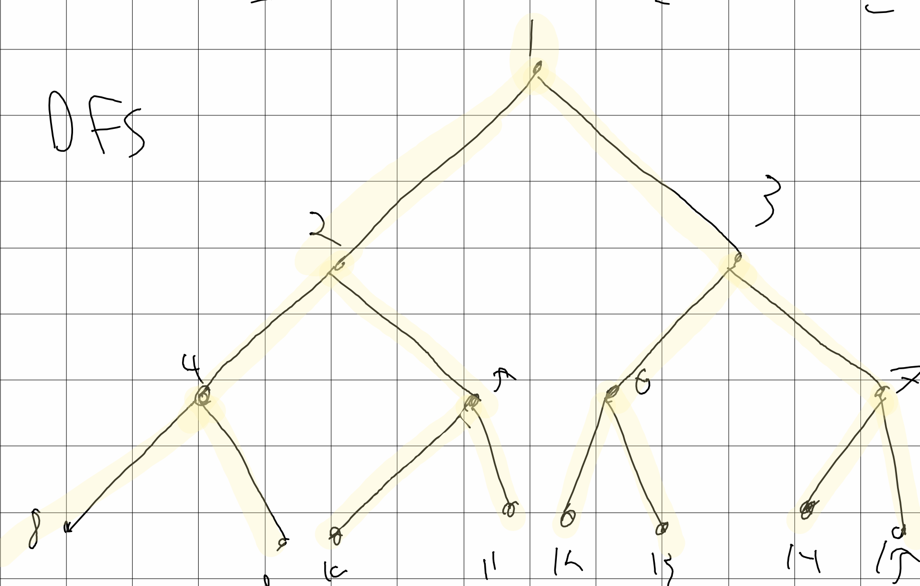
Apply BFS, DFS for trees

BFS



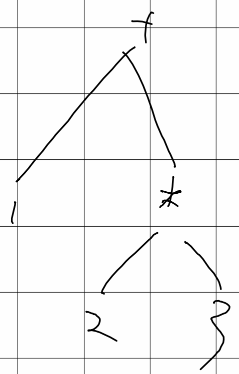
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

DFS



1
2
4
8
9
5
10
11
3
6
12
13
7
14
15

BFS / DFS tree → traverse vertices



1 + (2 * 3)

queue

BFS

lot of memory

entire frontier

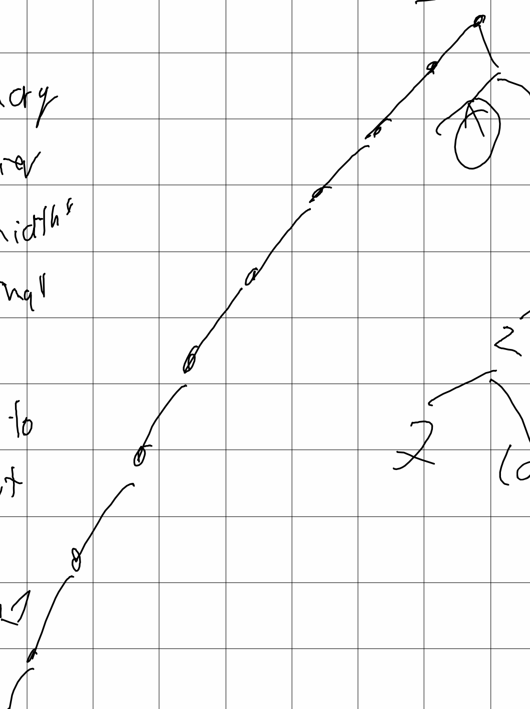
good if "width" graph is small

good if want to stay close to root

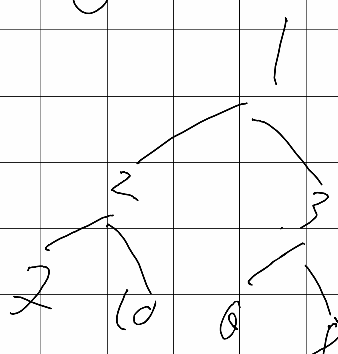
Remb. Dijkstra's w/edges weighted 1

stack

DFS

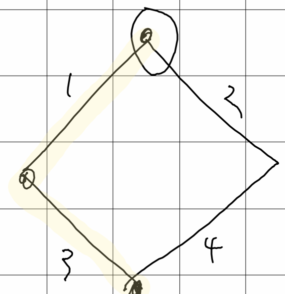


want to go far from root

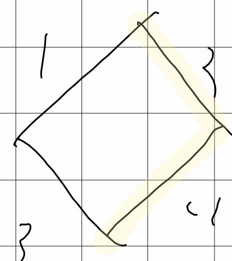


Min'l ~~length~~ weight Spanning tree

G weighted connected graph



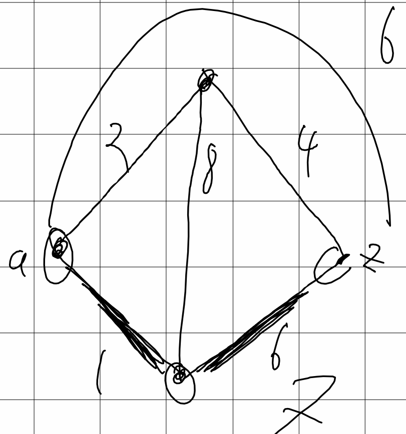
4



6

"Greedy"

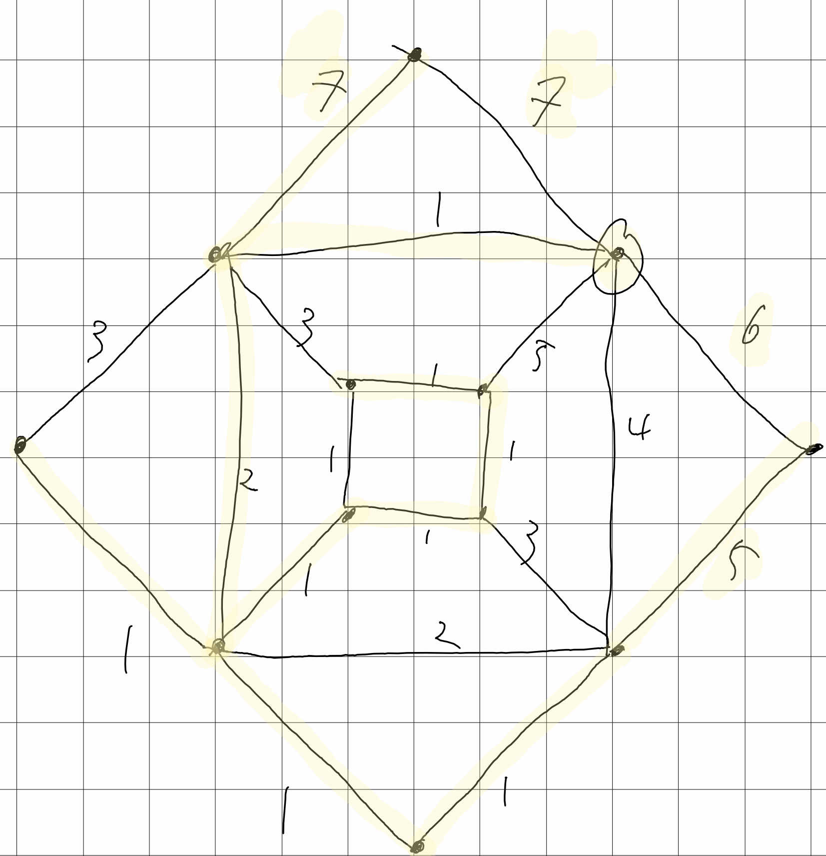
Start @ root,
pick smallest weighted edge
keep going until finish



Prim's algorithm.

"Greedy" algo to find mst

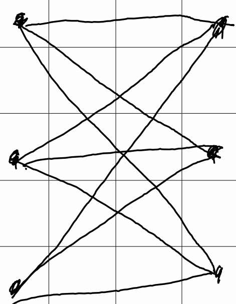
1. Start @ root
2. go through all edges in graph which have one vertex in T, one not in tree
3. Pick smallest weight such edge, add that to T
4. go back to step 2 until no more edges



Greedy algorithm works here!

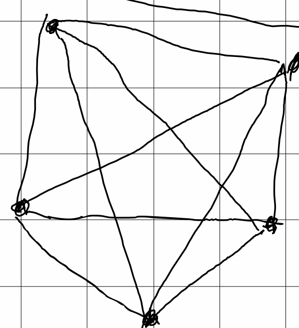
Planar

G is planar if you can draw it on paper without crossing edges
on the plane



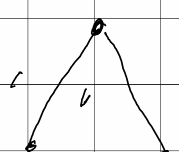
Thm (Kuratowski)
"only" counterexample.

G is planar iff it has no subgraph which reduces to $K_{3,3}$ or K_5



Planar graph $V - E + F = 2$

euler characteristic



$V = 3$
$E = 3$
$F = 1$

$3 - 3 + 2 = 2$

$K_{3,3}$ 6 vertices

9 edges

K_5 5 vertices

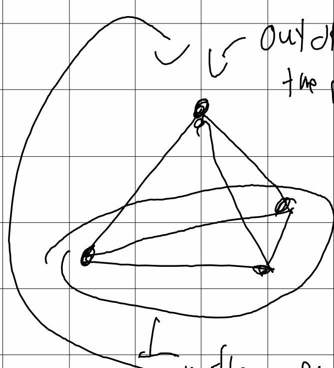
10 edges

< 5 vert → planar

< 9 edges → planar

5 vertices + 9 edges → planar

Is planar?

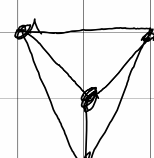


look @ this from above here

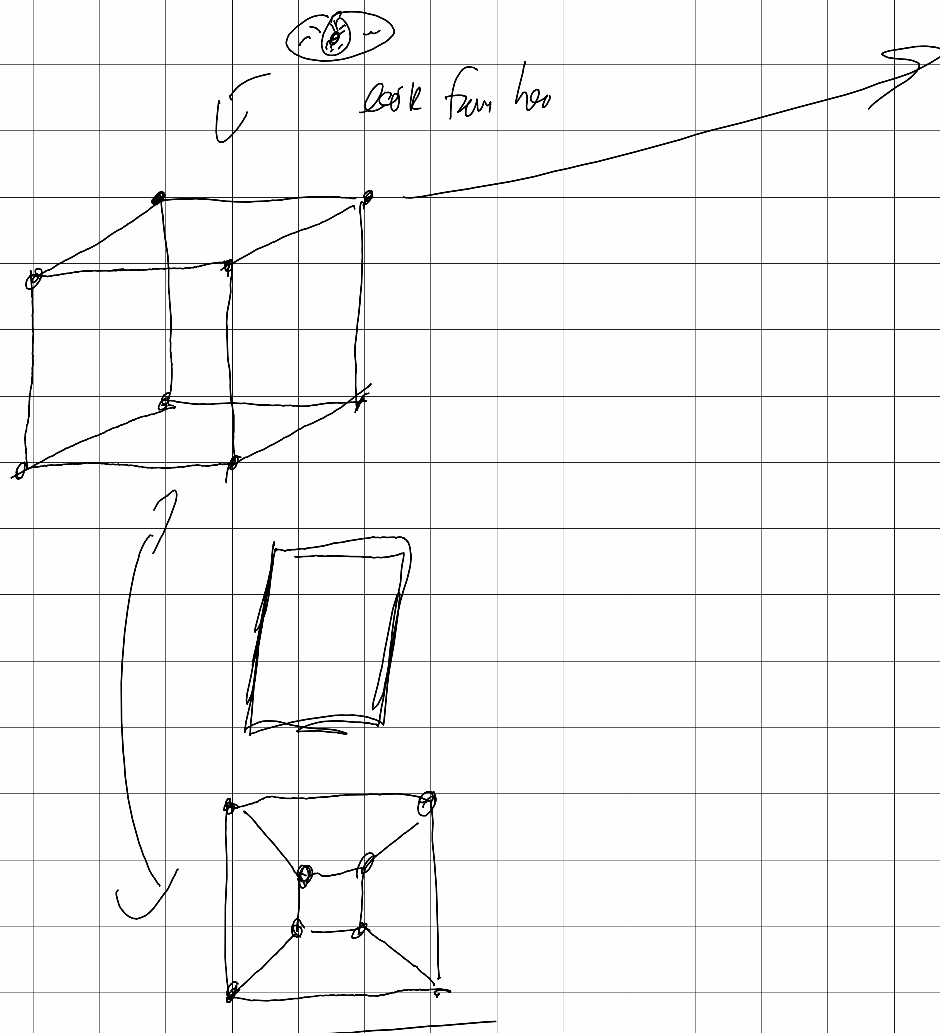
4 vert

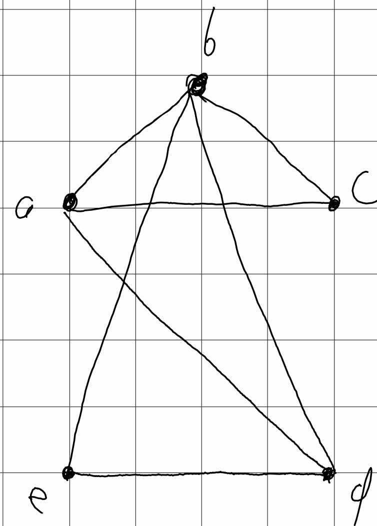
6 edges ⟹ planar

use 3d



top-down

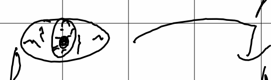

look from here



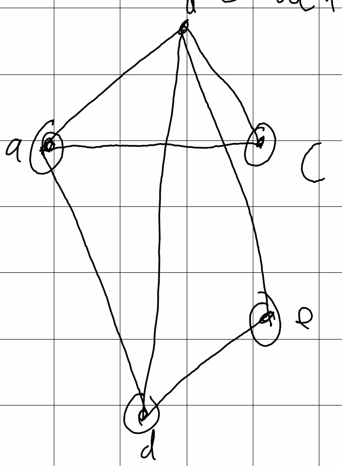
3 Δ's

abc
abd
bed

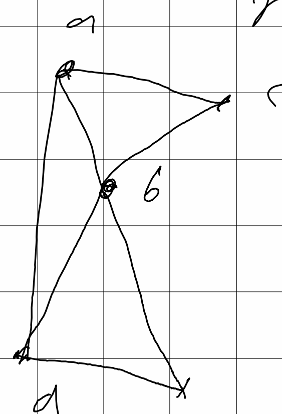
unfolding along $\overline{bd}$



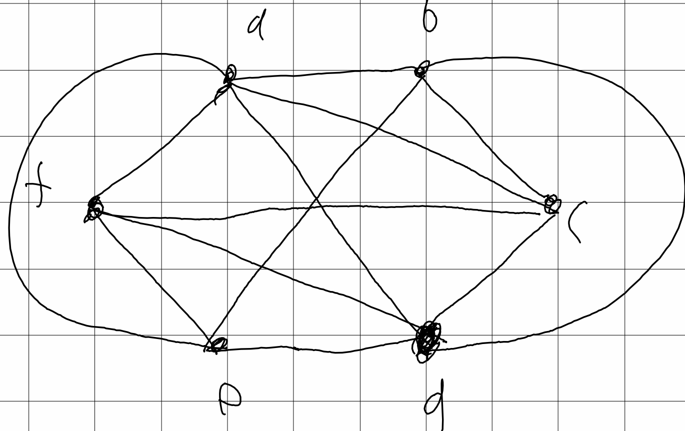
← abcd



planar drawing



0 → in plane



6 vertices

13 edges

$K_{3,3}$

	a	b	c	d	e	f
a	0	1	1	1	1	1
b	1	0	1	1	1	0
c	1	1	0	1	0	1
d	1	1	1	0	1	1
e	1	1	0	1	0	1
f	1	0	1	1	1	0

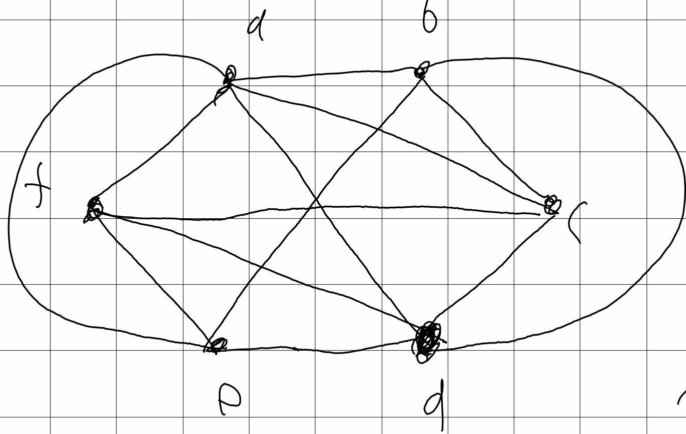
if $\{a,b\} \subseteq U_1$, $U_2 = \{c,d,e\}$

U_1	U_2
a	anything
b	acde
f	

$\{a,b,f\}$

cde

$K_{3,3}$



K_5?

5 vertices

x abcde

no ce

	a	b	c	d	e	f
a	0	1	1	1	1	1
b	1	0	1	1	1	0
c	1	1	0	1	0	1
d	1	1	1	0	1	1
e	1	1	0	1	0	1
f	1	0	1	1	1	0

x abcdf

no bf "

x abcef

ce

X abdef

no bf

X acdef

no ce

x bcdef

bf

no K_5