Oct 4, '20

Th – go over midterm

Counting edges

K_n = complete graph on n-vertices

one edge between all pairs of vertices

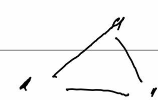
simple, undirected graph

K_1

$E_1 = 0$

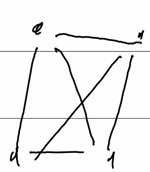
K_2

$E_2 = 1$



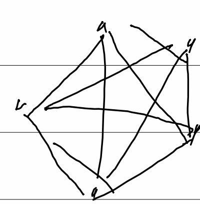
K_3

$E_3 = 3$



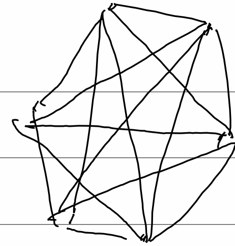
$E_4 = 6$

K_4



K_5

$E_5 = 10$



E_n = # edges in K_n

formula for E_n?

$n = 2, 3, 4, 5$

$E_n = \underbrace{1, 3}_{2}, \underbrace{3, 6}_{3}, \underbrace{6, 10}_{4}, \underbrace{10, 15}_{5}$

1, 2, 4, 8, 16

$10 \neq 1+3+6$

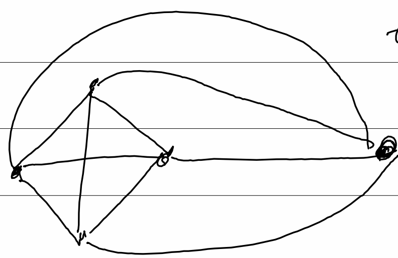
1, 2, 3, 4, 5

$6 \neq 3+1$

$3 \neq 1$

K_5

K_{4+1}

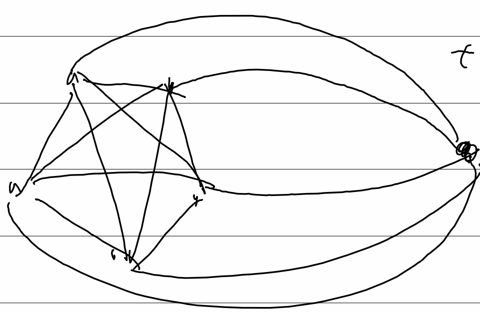


+4

$E_{n+1} = n + E_n$

$E_2 = 1$

K_6



+5

$E_1 = 0$

$E_2 = 1$

$E_{n+1} = n + E_n$

$1, 2, 4, 8, 16$

$a_n = 2a_{n-1}, \quad a_0 = 1$

$a_n = 2^n$

$E_n = ?$ not involving E_{n-1}

$$E_n = (n-1) + \underbrace{E_{n-1}}$$

$$= (n-1) + \big((n-2) + E_{n-2}\big)$$

$$= (n-1) + (n-2) + \big(\underbrace{(n-3)}_{n-1} + E_{n-3}\big)$$

$$= (n-1) + (n-2) + \cdots + \underbrace{1 + E_1}$$

$$= \sum_{k=1}^{n-1} k$$

$n = 2$

Prop. $E_n = \sum_{k=1}^{n-1} k$

Pf. induction

Gauss adding up $1+2+\dots+100$

$$E_n = \sum_{k=1}^{n-1} k$$

$$= \frac{n(n-1)}{2}$$

$$1 + 2 + 3 + 4 + \dots + (n-3) + (n-2) + (n-1)$$

$$1+(n-1) = n$$
$$+2+(n-2) = n$$
$$+3+(n-3) = n$$
$$+$$
$$\vdots$$

$$\frac{n-1}{2}$$

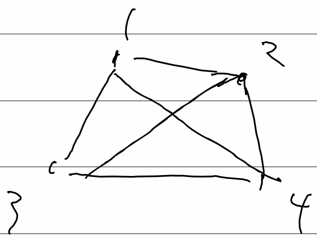
$$n\left(\frac{n-1}{2}\right)$$

$$E_n = \frac{n(n-1)}{2} = C(n,2) = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$$

$E_n = \underline{C(n,2)}$

K_n — vertices $\{1, \ldots n\}$

— edges <-> pairs of vertices



n people, all have to shake each other's hand

$$\sum_{k=1}^{n-1} k = C(n,2)$$

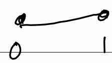
$\uparrow$ # edges in K_n

Q_n = hypercube of dimension n

$n=0$

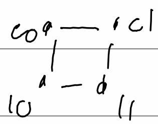
$E_0 = 0$

$n=1$



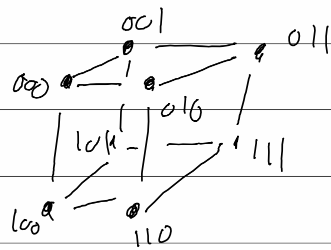
$E_1 = 1$

$n=2$



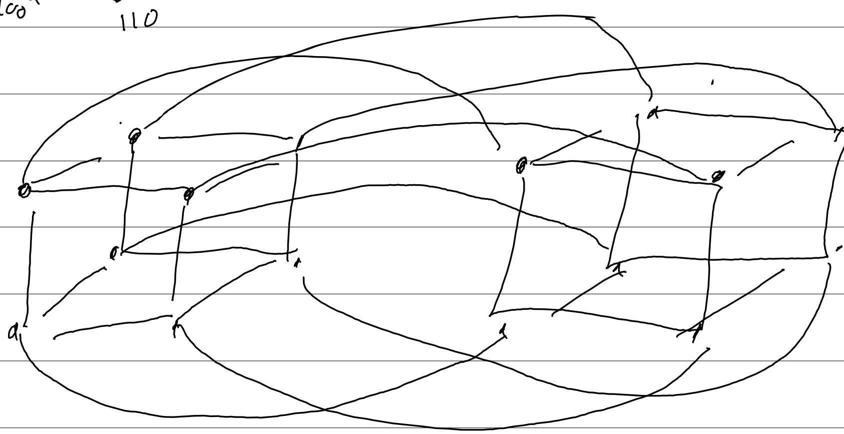
$E_2 = 4$

$n=3$



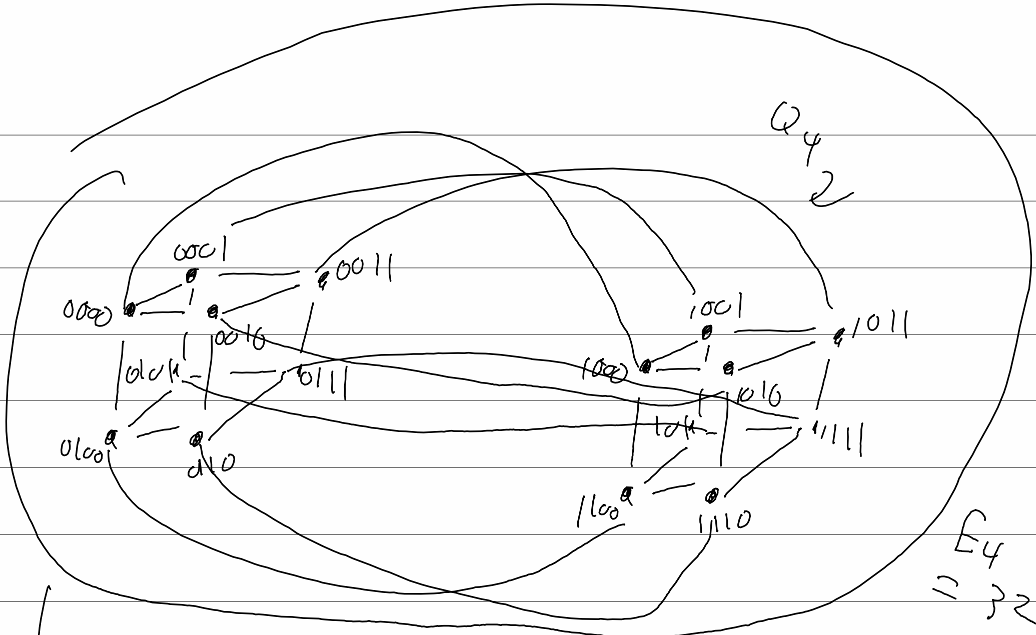
$E_3 = 12$

$n=4$



$Q_n = \begin{cases} \text{vertices} & n\text{-bit strings } 00\ldots01 \\ \text{edges - edge between 2 } n\text{-bit strings if they} \\ \quad \text{differ by one bit} \end{cases}$

differ in exactly one place



V_n = # vertices in Q_n

E_n = # edges in Q_n

V_n = # n-bit strings $= 2^n$

n	0	1	2	3	4
E_n	0	1	4	12	32

Prop. $E_n = n2^{n-1}$

3 → 4

$$Q_{n+1} = 0Q_n \qquad 1Q_n$$

$$E_{n+1} = E_n + E_n + V_n$$

$E_{n+1} = 2E_n + V_n$

$= 2E_n + 2^n$

prove $n2^{n-1}$ solves $E_{n+1} = 2E_n + 2^n$ given $E_1 = 1$

$E_n = 2E_{n-1} + 2^{n-1}$

$= 2(2E_{n-2} + 2^{n-2}) + 2^{n-1}$

$= 2^2 E_{n-2} + 2^{n-1} + 2^{n-1}$

$= 2^2(2E_{n-3} + 2^{n-3}) + 2^{n-1} + 2^{n-1}$

$= 2^3 E_{n-3} + 2^{n-1} + 2^{n-1} + 2^{n-1}$

$= \dots$

$= 2^k E_{n-k} + k2^{n-1}$

$k = (n-1)$

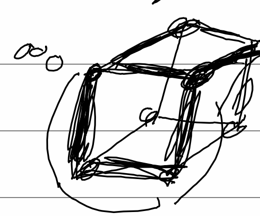
$= 2^{n-1} E_1 + (n-1)2^{n-1}$

$= n2^{n-1}$

each vertex in Q_n has n edges out of it

1001111 7 edges

$n2^n/2 = n2^{n-1}$



Hamming error correctio

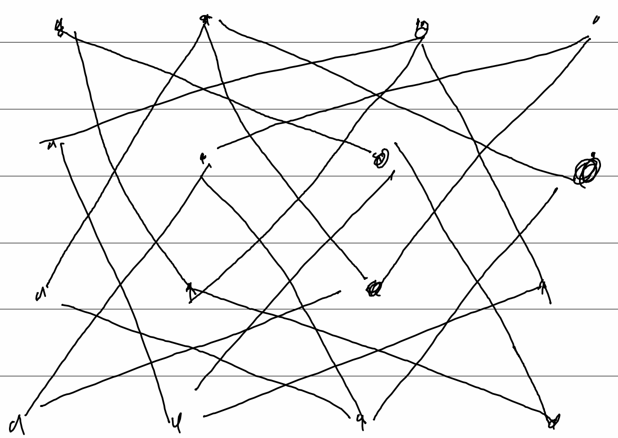
Hamming distance

Knight's graph

$V =$ $n \times n$-grid squares

pairs (i,j) $1 \le i,j \le n$

$E =$ 2 vert conn if they differ by a knight's move



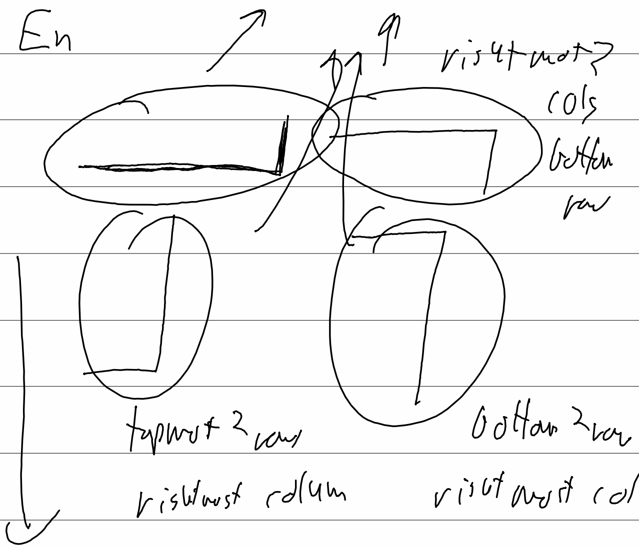
N_n, Hamiltonian path $\longleftrightarrow$ Knight's tour

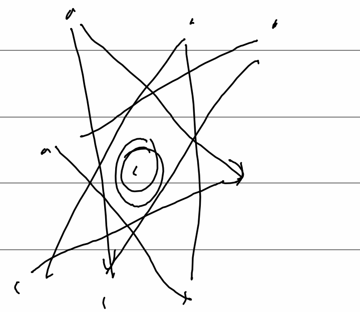
$n \geq 3$

$V_n = n^2$

$(n-2)(n-1)$

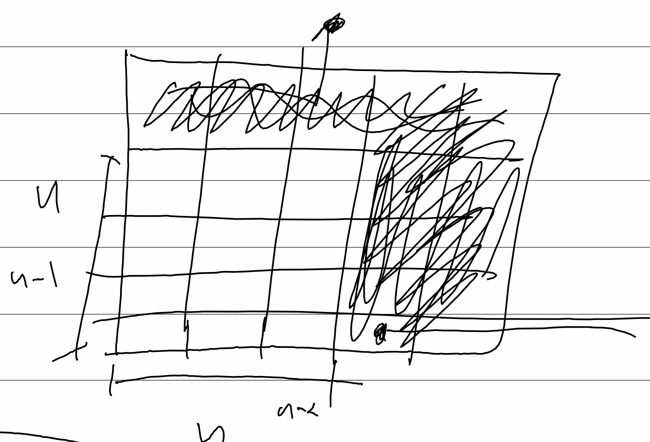
E_n





no Knight's tour!

\# vertices have this as a legal move?



$E_n = 4(n-1)(n-2)$

Inventory available/unavailable

Inventory w/ 87 items
either available or unavailable

50 available

wts. There are at least 2 available items which are <u>exactly</u> 7 places apart

A U A A U U U

A U

AAAAAAAAUUUUUUUU

AAAUUAA

$f(1) \quad f(2) \qquad f(7)$

$= 1 \quad = 2 \qquad = 6$

89 items,

50 available

A = set of available items

$$|A| \geq 50$$

$$F: A \longrightarrow \{1, \dots, 89\}$$

$$a \longmapsto \text{its position}$$

Goal: there are a, a' in A

$$f(a) - f(a') = 9$$

$$\cancel{A \times A \longrightarrow \{-88, \dots, 88\}}$$

$$\cancel{(a, b) \longmapsto f(a) - f(b)}$$

AAMUAAA
AAUUAA

label available items

$1, 2, \ldots, 50$

$f : \{1, \ldots, 50\} \longrightarrow \{1, \ldots, 89\}$

$f(i)$ = pos of item i

goal. $f(i) - f(j) = 9$ some $i \neq j$

$S_1 = \{ f(1), f(2), \ldots, f(50) \}$ all positions of avail item

$S_2 = \{ f(1)+9, f(2)+9, \ldots, f(50)+9 \}$ all poss of avail items shifted by 9

range $1, \ldots, 98$
holes
100 between them
n items

want to show $S_1 \cap S_2 \neq \phi$

let's say n in $S_1 \cap S_2$

$n \in S_1$ $\therefore$ $n = f(i)$ some i

$n \in S_2$ $\therefore$ $n = f(j) + 9$ some j

$f(i) = f(j) + 9$

paid 3 times

3rd form $\quad$ f(pigeon) the hole that this pigeon is in

$$x \longmapsto f(x)$$

$$X \xrightarrow{f} Y$$

pigeons $\quad$ holes

$|X| = n, \quad |Y| = m$

there are $\lceil \frac{n}{m} \rceil$ distinct elts of X

$x_1, \ldots, x_{\lceil \frac{n}{m} \rceil}$

s.t. $f(x_1) = \ldots = f(x_{\lceil \frac{n}{m} \rceil})$

$n > m \quad \lceil \frac{n}{m} \rceil \geq 2$

$n = 17 \qquad \lceil \frac{17}{5} \rceil = 4$

$m = 5$