off today 12

A, B sets

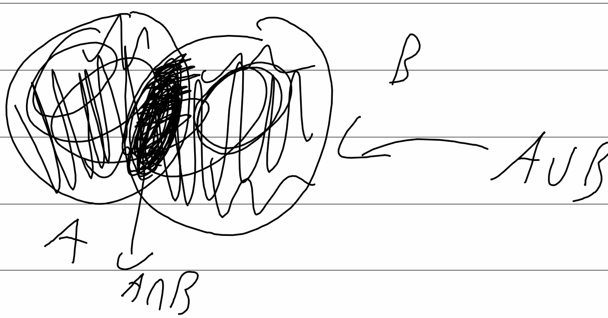
inclusive, not exclusive

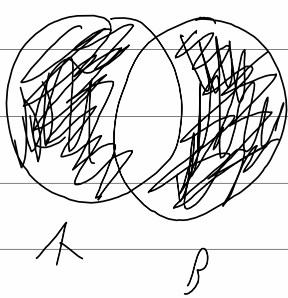
either

$A \cup B = \{x \mid x \in A$ or $x \in B\}$

p or q

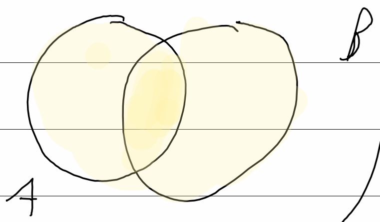
$A \cap B = \{x \mid x \in A$ AND $x \in B\}$



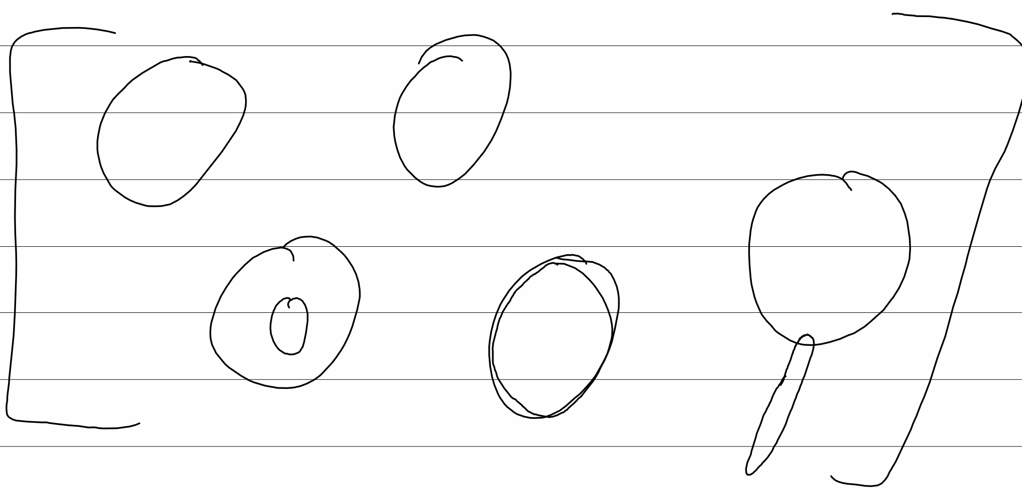


Thm 1.1.22

$A \cap B \subseteq A \cup B$?



Likely true



$\boxed{A \cap B \subseteq A \cup B}$

P AND Q $\Rightarrow$ P

Pf. Let $x \in A \cap B$,

Goal is to show that $x \in A \cup B$.

($x \in A \cap B \Rightarrow x \in A \cup B$)

know: By definition of $A \cap B$, this means that

$\boxed{\underset{\text{true}}{\underline{x \in A}} \text{ AND } \underset{\text{true}}{\underline{x \in B}}}$

seek: Goal. To show $x \in A \cup B$

By definition of $A \cup B$, we want to show that

$\boxed{x \in A}$ OR $\boxed{x \in B}$

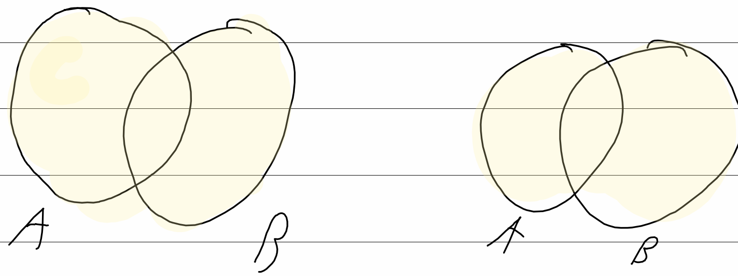
Know $x \in A$ AND $x \in B$, so we have in particular that $\underline{x \in A}$.

Hence, $\underline{x \in A \text{ or } x \in B}$. That is, $x \in A \cup B$.

So we have shown $x \in A \cap B \Rightarrow x \in A \cup B$.

By definition, $A \cap B \subseteq A \cup B$.

Problem. $A \cup B = B \cup A$



$A \cup B$ $\qquad\qquad$ $B \cup A$

Pf. suffices to show $A \cup B \subseteq B \cup A$ (1) . $B \cup A \subseteq A \cup B$ (2)

(1) $A \cup B \subseteq B \cup A$?

Let $x \in A \cup B$

By def, $x \in A$ or $x \in B$ of $A \cup B$

i) $\qquad$ ii)

i) $x \in A$. Then ($x \in B$ OR $x \in A$)

Thus, $x \in B \cup A$ by def'n of $B \cup A$

ii) $x \in B$ then ($x \in B$ or $x \in A$)

Thus $x \in B \cup A$ by def'n of $B \cup A$

We showed $x \in A \cup B \implies x \in B \cup A$, i.e. $A \cup B \subseteq B \cup A$ as desired.

(2) Same but flip A, B

Conclusion. We've shown $A \cup B \subseteq B \cup A$

and $B \cup A \subseteq A \cup B$

Thus, $A = B$.

$2 + 7/3$

Problem. $7^n - 1$ is divisible by 6 for all $n \geq 1$.

Pf. We proceed by induction on the sentence

$p(n) =$ " $7^n - 1$ is divisible by 6"

Base case. wts $p(1) =$ " $7^1 - 1$ is divisible by 6"

$7^1 - 1 = 6$

6 is divisible by 6, $6 = 6 \cdot 1$

Thus, $p(1)$ holds

Ind. step. Suppose $p(n)$ holds for some $n \geq 1$.

we want to show $p(n+1)$ holds

$p(n) =$ " $\underline{7^n - 1}$ is divisible by 6"

wts $7^{n+1} - 1$ is divisible by 6

$$\begin{aligned} 7^{n+1} - 1 &= 7 \cdot 7^n - 1 \\ &= (6+1) 7^n - 1 \\ &= 6 \cdot 7^n + 7^n - 1 \end{aligned}$$

$6 \cdot 7^n$ is divisible by 6

$7^n - 1$ is divisible by 6 by <u>inductive hypothesis</u>

$P \Rightarrow (Q \text{ or } R)$

That is, there exists integer m s.t.

$7^{n} - 1 = 6m$

$$7^{n+1} - 1 = 6 \cdot 7^{n} + 7^{n} - 1$$
$$= 6 \cdot 7^{n} + 6m$$
$$= 6(7^{n} + m)$$

Thus, $7^{n+1} - 1$ is divisible by 6

So we conclude by mathematical induction, that $7^{n} - 1$ div. by 6 for all $n \geq 1$

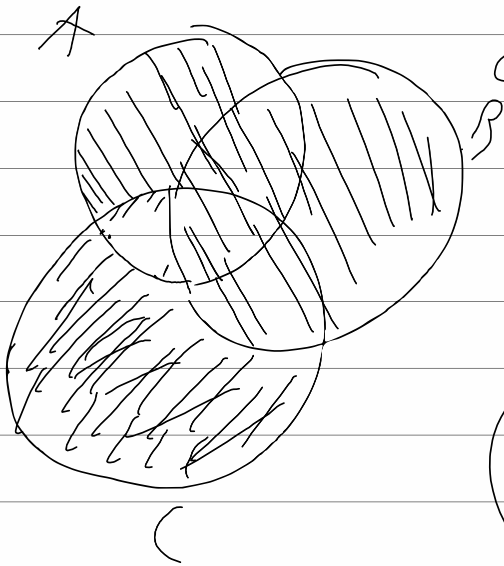
$A \cup B - (C - B) = (A \cup B - C) + B$

$\cup$

$A \cup B - (C - B)$

$=$

$(A \cup B - C) \cup B$



$\cancel{A \cup B - (C - B)}$

$A \cup B - (C - B)$

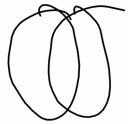
$=$

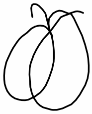
$((A \cup B) - C) \cup B$

$(A \cup B - C) \cup B$

$(A \cup B)$

$- (C - B)$





$x \in A \cup B$

$[x \in A$ or $x \in B]$

$\Downarrow$

$x \in B \cup A$

$\Updownarrow$

$[x \in B$ or $x \in A$

$\cancel{A \cup B - (C - B)}$